Midterm: Some problems for review

The exam will be held in class (Badgley 146) during the class time (9 - 9:50 AM) on Thursday November 3. The material covered is Section 1.1 - 2.4. It is a closed book exam. A single sided, handwritten, 3" x 5" note card is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.

You should review the homework problems, worksheet problems, and read examples in the textbook. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Find the domain of a function.
- Find the limit of a function using limit rule: plug-in, sum, product, quotient, Squeeze Theorem.
- Show that a limit does not exist. This could be done by showing that the left and right limits do not agree, or that the function blows up near the limit point, or that the function oscillates near the limit point.
- Use the Intermediate Value Theorem to show that an equation has a solution.
- Find the vertical and horizontal asymptotes of a function.
- Find derivatives using the definition (as the limit of a difference quotient).
- Find derivatives using differentiation rules.
- Find the instantaneous rate of change.
- Find tangent line to a given curve at a given point.

Additional problems to practice:

1) Find the domain of $f(x) = \sqrt{16 - x^4}$.

2) Sketch the function

$$f(x) = \begin{cases} 1 + x & \text{if} \quad x < 0, \\ 1 + x^2 & \text{if} \quad x \geq 0. \end{cases}$$

3) Find the following limits:

(a) $\lim\limits_{x \to 0} \cos(x + \sin x)$

(b) $\lim\limits_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 3}$

(c) $\lim\limits_{x \to 1^+} \frac{x^2 - 9}{x^2 + 2x - 3}$

(d) $\lim\limits_{x \to \infty} \frac{x^2 - 9}{x^2 + 2x - 3}$

(e) $\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x+1}$

(f) $\lim_{x \to 1} \frac{x^2-1}{x^2+2x-3}$

(g) $\lim_{t \to 0} \frac{t^3}{\tan^3(2t)}$

(h) $\lim_{x \to \infty} \frac{\sin x}{\sqrt{x}}$

4) Show that the equation $\cos x = x$ has a solution.

5) Determine whether the function f given below is differentiable at 0.

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if} \quad x \neq 0, \\ 0 & \text{if} \quad x = 0. \end{cases}$$

6) Find $f'(x)$ and $f'(1)$ for the following functions:

(a) $f(x) = x + \frac{1}{x}$

(b) $f(x) = \frac{1-x}{2+x}$

(c) $f(x) = \frac{\sin x}{x^2}$

7) At what point(s) does the curve $y = 2x^3 + 9x^2 - 24x + 6$ has a peak or valley?

8) Find the slope of the tangent line to the curve $y = \frac{1}{x}$ at point $(2, \frac{1}{2})$.

9) Find all points on the curve $y = \frac{1}{x}$ such that the tangent line to the curve at those points is parallel to the line $y = -2x$.

Answer keys:

1) $[-2, 2]$

2)

3) a. 1
   b. 0
   c. $-\infty$
   d. 1
   e. -1
   f. $\frac{1}{2}$
   g. $\frac{1}{8}$
   h. 0

4)

5) No, f is not differentiable at 0 because $f'(0) = \lim \ldots$ does not exist.

6) a. $f'(x) = 1 - \frac{1}{x^2}$ and $f'(1) = 0$.
   b. $f'(x) = \frac{-3}{(2+x)^2}$ and $f'(1) = -1/3$.
   c. $f'(x) = \frac{x^2 \cos x - 2x \sin x}{x^4}$ and $f'(1) = \cos 1 - 2 \sin 1$.

7) Everywhere in $\mathbb{R}^2$ except for the x-axis and the y-axis

8) At $(1, -7)$ and $(-4, 118)$

9) $-1/4$

10) $(\frac{1}{\sqrt{2}}, \sqrt{2})$ and $(-\frac{1}{\sqrt{2}}, -\sqrt{2})$