MATH 251, MIDTERM, FALL 2022

INSTRUCTOR: TUAN PHAM

Name

Instructions:

- This is a closed-book exam, 50 minutes long.
- A single sided, handwritten, 3" x 5" note card is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.
- For Problems 1-7, fill in the bubbles on this front page. To each problem, only one answer is correct.
- For Problems 8, 9 and 10, make sure to show all necessary steps. Mysterious answers will receive little or no credit.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ
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Problem	Possible points	Earned points
1-7	14	
8	5	
9	5	
10	5	
Total	29	

Problem 1. (2 points) Let $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$. Which of the following is the composite function $f \circ g$? That is, function $f(g(x))$.

A. $\frac{1}{x^2+1}$

B. $\frac{2}{x^2}$

C. $\frac{1}{x^2} + 1$

D. $\frac{1}{(x+1)^2}$

Problem 2. (2 points) Suppose a function f is not defined at $x = a$. Which of the following statements is false?

A. f is not continuous at a.

B. f is not differentiable at a.

C. $\lim_{x \to a} f(x)$ does not exist.

Problem 3. (2 points) If the $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$ then f is discontinuous at a. True or false?

A. True
B. False

Problem 4. (2 points) Choose the correct value of the limit

$$\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 + 1}}$$

A. 1

B. -1

C. ∞

D. $-\infty$

Problem 5. (2 points) Choose the correct value of the limit

$$\lim_{x \to \infty} \frac{x(2x^2 - 3x + 5)}{(x^2 + 1)(x + 1)}$$

A. 1

B. 2

C. 0

D. ∞

Problem 6. (2 points) Let $f(x) = x + \frac{x}{x+1}$. Find $f'(1)$.

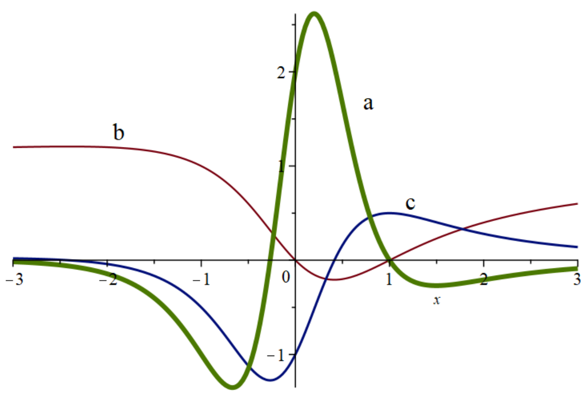
A. $-1/4$

B. $1/4$

C. $3/4$

D. $5/4$

Problem 7. (2 points) The figure below contains the graphs of f, f', and f''. The graphs of these functions in that order are



A. a, b, c

B. a, c, b

C. b, a, c

D. b, c, a

Problem 8. (5 points) Evaluate the polynomial $x^3 - 3x + 1$ at $x = -2, -1, 0, 1, 2$ and explain why it has three distinct roots.

$f(x) = x^3 - 3x + 1$ is a continuous function.

$f(-2) = -1 < 0$, $f(-1) = 1 > 0$ } there is a root between -2 and -1.

$f(0) = 0 > 0$, $f(1) = -1 < 0$ } there is a root between 0 and 1.

$f(1) = -1 < 0$, $f(2) = 3 > 0$ } there is a root between 1 and 2.

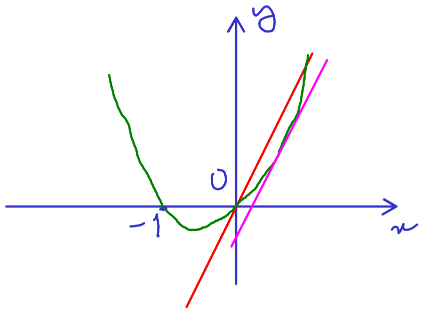
By Intermediate value theorem, f has a root in $(-2,-1)$, a root in $(0,1)$, and a root in $(1,2)$. Therefore, it has three distinct roots.

Problem 9. (5 points) Use the limit laws you learned to find the limit

$$\lim_{x\to -1} \frac{x^2-2x-3}{x^2+4x+3}$$

$$\frac{x^2-2x-3}{x^2+4x+3} = \frac{(x+1)(x-3)}{(x+1)(x+3)} = \frac{x-3}{x+3} \xrightarrow{x\to -1} \frac{-1-3}{-1+3} = -2.$$

Problem 10. (5 points) Find the point on the parabola $y = x^2 + x$ at which the tangent line to the parabola is parallel to the line $y = 3x$. What is the equation for the tangent line at that point?



$f(x) = x^2 + x$

$f'(x) = 2x + 1$ is the slope of the tangent line to the graph of f at $(x, f(x))$.

For the tangent line to be parallel to the line $y = 3x$, the two slopes must be equal. That is, $f'(x) = 3$ which leads to $x = 1$.

At $x = 1$, $f(x) = f(1) = 2$. At the point $(1, 2)$, the tangent line to the parabola is parallel to $y = 3x$. The equation of the tangent line is

$$y - 2 = 3(x - 1)$$

or equivalently, $y = 3x - 1$.