Lecture 14

Wednesday, October 4, 2023 10:52 AM

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Compound interest
- one-time payment : $P(1+i)^m$
- periodic payment : $p\,\dfrac{(1+i)^m - 1}{i}$

Consider the following situation: you put \$50 in the bank at the end of each month and the bank gives a monthly interest rate at 0.5%.

After month 1, you have 50 in the account

After month 2, you have $50 + 50(1+0.5\%)$

After month 3, you have $50 + 50(1+0.5\%) + 50(1+0.5\%)^2$

After month 4, you have $50 + 50(1+0.5\%) + 50(1+0.5\%)^2 + 50(1+0.5\%)^3$

After month n, you have $50 + 50(1+0.5\%) + 50(1+0.5\%)^2 + \dots + 50(1+0.5\%)^{n-1}$

To generalize the problem, we denote

$p =$ monthly payment

$i =$ interest rate per period

The accumulated balance after n periods is

$$p + p(1+i) + p(1+i)^2 + p(1+i)^3 + \dots + p(1+i)^{n-1}$$

$$= p\,\frac{(1+i)^n - 1}{i}$$

Ex two methods of investment:

(1) Put \$100 in the bank each month at APR = 7.2%

(2) Put \$20,000 in the bank (one time payment) at APR = 6%

Which method is a better investment in 5 years? 30 years?

- In 5 years: the accumulated balance of the first method is

$$100 \cdot \frac{\left(1+\frac{0.072}{12}\right)^{60}-1}{\frac{0.072}{12}} \approx$$

Second method:

$$20000\left(1+\frac{0.06}{12}\right)^{60} \approx$$

So, the second method is better.

- In 30 years, the first method gives

$$100 \cdot \frac{\left(1+\frac{0.072}{12}\right)^{360}-1}{\frac{0.072}{12}} \approx$$

The second method gives

$$20000\left(1+\frac{0.06}{12}\right)^{360} \approx$$

So the first method is better.

*Observation: the monthly investment is better in a long run. But in a short run, the one-time investment is better.