Worksheet 11/22/2023

Law of Large Numbers: Consider an event A with probability P(A) in a single trial. For a large number of trials, the proportion of trials in which event A occurs will be close to the probability P(A). The larger the number of trials, the closer the proportion should be to P(A).

Random variable: a *quantity* that can assume different values in different events.

Expected value: the long-run average value of a random variable.

The expected value of a random variable X is calculated as follows:

$$EX = v_1 p_1 + v_2 p_2 + \ldots + v_n p_n$$

where $p_1, p_2, \ldots, p_n$ are probabilities (adding up to 1) of events 1, 2, …, n, and $v_1, v_2, \ldots, v_n$ are the values of the random variable X in events 1, 2,…, n.

Gambler's fallacy: (also called *gambler's ruin*) the mistaken belief that a streak of bad luck makes a person due for a streak of good luck (or that a streak of good luck will continue).

House edge: For any particular game, the house edge is the expected value *to the house* (company/casino) of each individual bet.

1) An insurance policy sells for $325. Based on past data, an average of 1 in 100 policyholders will file a $10,000 claim, an average of 1 in 250 policyholders will file a $25,000 claim, and an average of 1 in 500 policyholders will file a $50,000 claim. Find the expected value (to the company) per policy sold. Find the expected profit if the company sells 1000 polices and if it sells 100,000 policies. Explain whether this profit can reasonably be "expected" in each case.

2) The price of a ticket is $1 and there is a 1 in 10 probability of winning $1, a 1 in 50 probability of winning $5, a 1 in 500 probability of winning $100, and a 1 in 1 million probability of winning $100,000. Find the expected value (to you) of a single ticket. Find the average winnings or loss expected if you purchase 1000 tickets. Explain your answers.

3) Suppose you play a coin toss game in which you win \$1 if a head appears and lose \$1 if a tail appears. In the first 100 coin tosses, heads comes up 46 times and tails comes up 54 times. What percentage of times has heads come up in the first 100 tosses? What is your net gain or loss at this point?

Suppose you toss the coin 200 more times (a total of 300 tosses), and at that point heads has come up 47% of the time. Is this increase in the percentage of heads consistent with the law of large numbers? What is your net gain or loss at this point?

How many heads would you need in the next 100 tosses in order to break even after 400 tosses? Is it reasonable to expect this to occur?

Suppose that, still behind after 400 tosses, you decide to keep playing because you are "due" for a winning streak. Explain how this belief would illustrate the gambler's fallacy.