Worksheet 11/27/2023

Arrangement with repetition: from n items, the number of arrangements of r items, allowing repetition, is n^r.

Permutation: from n items, the number of arrangements of r items, not allowing repetition, is

$$\frac{n!}{(n-r)!} = n \times (n\ -\ 1) \times \ldots \times (n\ -\ r + 1)$$

Combination: from n items, the number of arrangements of r items, disregarding order and not allowing repetition, is

$$\frac{n!}{(n-r)!\,r!} = \frac{n \times (n\ -\ 1) \times \ldots \times (n\ -\ r + 1)}{1 \times 2 \times \ldots \times r}$$

1) Calculate

$8!$ $\qquad$ $\frac{9!}{4!}$ $\qquad$ $\frac{10!}{3!\,7!}$

2) How many different seven-digit phone numbers can be formed?

3) How many different five-character passwords can be formed from the lowercase letters of the alphabet if repetition is not allowed?

4) A city council with eight members must elect a four-person executive committee consisting of a mayor, deputy mayor, secretary, and treasurer. How many executive committees are possible?

5) How many 6-person lineups can be formed from a 10-player volleyball roster? (In volleyball, every player plays every position.)

6) How many different telephone numbers of the form *aaa-bbb-cccc* can be formed if the area code *aaa* can only begin with the numbers 2 through 7 and the exchange *bbb* cannot begin with 0?

7) Find the probability of being dealt a 10, jack, queen, king, and ace, all of the same suit, from a standard 52-card deck.

8) The U.S. Postal Service uses both five-digit and nine-digit ZIP codes.

(a) How many five-digit ZIP codes are available to the U.S. Postal Service?

(b) For a U.S. population of 300 million people, what is the average number of people per five-digit ZIP code if all possible ZIP codes are used?

(c) How many nine-digit ZIP codes are available to the U.S. Postal Service? Could everyone in the United States have his or her own personal nine-digit ZIP code?