Worksheet 11/3/2023

Normal distribution: a special symmetric, bell-shaped distribution, with a single peak. The normal distribution is completely identified by its mean and standard deviation.

68-95-99.7 rule: about 68%, 95%, 99.7% of data points fall within 1, 2, 3 standard deviations of the mean, respectively

Standard score (z-score) of a data value = the number of standard deviations from the mean

$$z = \frac{data\ value\ -\ mean}{standard\ deviation}$$

Percentile of a data value = the percentage of all data points that are less than or equal to it.

Scores on the verbal section of the GRE exam have a mean of 150 and a standard deviation of 8.5. Scores on the quantitative section of the exam have a mean of 152 and a standard deviation of 8.9. Assume the scores are normally distributed.

- 1) Suppose a graduate school requires (among other qualifications) that applicants score at or above the 90th percentile on both exams. What verbal and quantitative scores are required?
- 2) What percentage of students taking the verbal exam score above 160?
- 3) What percentage of students taking the quantitative exam score above 160?
- 4) Scores on the quantitative exam range between 130 and 170. What are the percentiles of these scores?
- 5) Scores on the verbal exam range between 130 and 170. What are the percentiles of these scores?

6) Students intending to study arts and humanities in graduate school have a mean score of 150 on the quantitative exam and a mean score of 157 on the verbal exam. Find the percentile of 150 on the quantitative exam. Find the percentile of 157 on the verbal exam.

| z-score | Percentile | z-score | Percentile | z-score | Percentile | z-score | Percentile |
|---------|------------|---------|------------|---------|------------|---------|------------|
| -3.5 | 0.02 | -1.0 | 15.87 | 0.0 | 50.00 | 1.1 | 86.43 |
| -3.0 | 0.13 | -0.95 | 17.11 | 0.05 | 51.99 | 1.2 | 88.49 |
| -2.9 | 0.19 | -0.90 | 18.41 | 0.10 | 53.98 | 1.3 | 90.32 |
| -2.8 | 0.26 | -0.85 | 19.77 | 0.15 | 55.96 | 1.4 | 91.92 |
| -2.7 | 0.35 | -0.80 | 21.19 | 0.20 | 57.93 | 1.5 | 93.32 |
| -2.6 | 0.47 | -0.75 | 22.66 | 0.25 | 59.87 | 1.6 | 94.52 |
| -2.5 | 0.62 | -0.70 | 24.20 | 0.30 | 61.79 | 1.7 | 95.54 |
| -2.4 | 0.82 | -0.65 | 25.78 | 0.35 | 63.68 | 1.8 | 96.41 |
| -2.3 | 1.07 | -0.60 | 27.43 | 0.40 | 65.54 | 1.9 | 97.13 |
| -2.2 | 1.39 | -0.55 | 29.12 | 0.45 | 67.36 | 2.0 | 97.72 |
| -2.1 | 1.79 | -0.50 | 30.85 | 0.50 | 69.15 | 2.1 | 98.21 |
| -2.0 | 2.28 | -0.45 | 32.64 | 0.55 | 70.88 | 2.2 | 98.61 |
| -1.9 | 2.87 | -0.40 | 34.46 | 0.60 | 72.57 | 2.3 | 98.93 |
| -1.8 | 3.59 | -0.35 | 36.32 | 0.65 | 74.22 | 2.4 | 99.18 |
| -1.7 | 4.46 | -0.30 | 38.21 | 0.70 | 75.80 | 2.5 | 99.38 |
| -1.6 | 5.48 | -0.25 | 40.13 | 0.75 | 77.34 | 2.6 | 99.53 |
| -1.5 | 6.68 | -0.20 | 42.07 | 0.80 | 78.81 | 2.7 | 99.65 |
| -1.4 | 8.08 | -0.15 | 44.04 | 0.85 | 80.23 | 2.8 | 99.74 |
| -1.3 | 9.68 | -0.10 | 46.02 | 0.90 | 81.59 | 2.9 | 99.81 |
| -1.2 | 11.51 | -0.05 | 48.01 | 0.95 | 82.89 | 3.0 | 99.87 |
| -1.1 | 13.57 | 0.0 | 50.00 | 1.0 | 84.13 | 3.5 | 99.98 |