Worksheet 11/3/2023
Normal distribution: a special symmetric, bell-shaped distribution, with a single peak. The normal distribution is completely identified by its mean and standard deviation.

68-95-99.7 rule: about $68 \%, 95 \%, 99.7 \%$ of data points fall within $1,2,3$ standard deviations of the mean, respectively

Standard score (z-score) of a data value $=$ the number of standard deviations from the mean

$$
z=\frac{\text { data value }- \text { mean }}{\text { standard deviation }}
$$

Percentile of a data value $=$ the percentage of all data points that are less than or equal to it.
Scores on the verbal section of the GRE exam have a mean of 150 and a standard deviation of 8.5. Scores on the quantitative section of the exam have a mean of 152 and a standard deviation of 8.9. Assume the scores are normally distributed.

1) Suppose a graduate school requires (among other qualifications) that applicants score at or above the 90th percentile on both exams. What verbal and quantitative scores are required?
2) What percentage of students taking the verbal exam score above 160 ?
3) What percentage of students taking the quantitative exam score above 160 ?
4) Scores on the quantitative exam range between 130 and 170. What are the percentiles of these scores?
5) Scores on the verbal exam range between 130 and 170. What are the percentiles of these scores?
6) Students intending to study arts and humanities in graduate school have a mean score of 150 on the quantitative exam and a mean score of 157 on the verbal exam. Find the percentile of 150 on the quantitative exam. Find the percentile of 157 on the verbal exam.

| $z$-score | Percentile | $z$-score | Percentile | $z$-score | Percentile | $z$-score | Percentile |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.5 | 0.02 | -1.0 | 15.87 | 0.0 | 50.00 | 1.1 | 86.43 |
| -3.0 | 0.13 | -0.95 | 17.11 | 0.05 | 51.99 | 1.2 | 88.49 |
| -2.9 | 0.19 | -0.90 | 18.41 | 0.10 | 53.98 | 1.3 | 90.32 |
| -2.8 | 0.26 | -0.85 | 19.77 | 0.15 | 55.96 | 1.4 | 91.92 |
| -2.7 | 0.35 | -0.80 | 21.19 | 0.20 | 57.93 | 1.5 | 93.32 |
| -2.6 | 0.47 | -0.75 | 22.66 | 0.25 | 59.87 | 1.6 | 94.52 |
| -2.5 | 0.62 | -0.70 | 24.20 | 0.30 | 61.79 | 1.7 | 95.54 |
| -2.4 | 0.82 | -0.65 | 25.78 | 0.35 | 63.68 | 1.8 | 96.41 |
| -2.3 | 1.07 | -0.60 | 27.43 | 0.40 | 65.54 | 1.9 | 97.13 |
| -2.2 | 1.39 | -0.55 | 29.12 | 0.45 | 67.36 | 2.0 | 97.72 |
| -2.1 | 1.79 | -0.50 | 30.85 | 0.50 | 69.15 | 2.1 | 98.21 |
| -2.0 | 2.28 | -0.45 | 32.64 | 0.55 | 70.88 | 2.2 | 98.61 |
| -1.9 | 2.87 | -0.40 | 34.46 | 0.60 | 72.57 | 2.3 | 98.93 |
| -1.8 | 3.59 | -0.35 | 36.32 | 0.65 | 74.22 | 2.4 | 99.18 |
| -1.7 | 4.46 | -0.30 | 38.21 | 0.70 | 75.80 | 2.5 | 99.38 |
| -1.6 | 5.48 | -0.25 | 40.13 | 0.75 | 77.34 | 2.6 | 99.53 |
| -1.5 | 6.68 | -0.20 | 42.07 | 0.80 | 78.81 | 2.7 | 99.65 |
| -1.4 | 8.08 | -0.15 | 44.04 | 0.85 | 80.23 | 2.8 | 99.74 |
| -1.3 | 9.68 | -0.10 | 46.02 | 0.90 | 81.59 | 2.9 | 99.81 |
| -1.2 | 11.51 | -0.05 | 48.01 | 0.95 | 82.89 | 3.0 | 99.87 |
| -1.1 | 13.57 | 0.0 | 50.00 | 1.0 | 84.13 | 3.5 | 99.98 |

