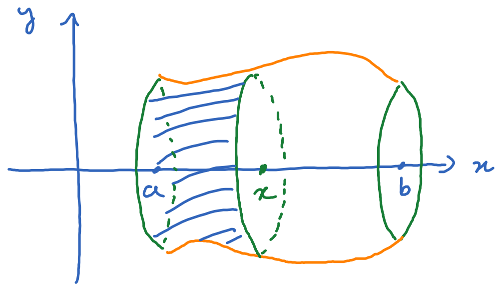
Lecture 23

Wednesday, October 4, 2023 1:25 AM

* Prayer
* Surface of revolution



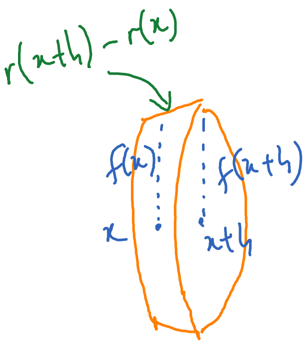
$S(x)$ = area of the surface of revolution between the cross sections at a and x.

Thus, $S(a) = 0$.

We want to find $S(b)$.

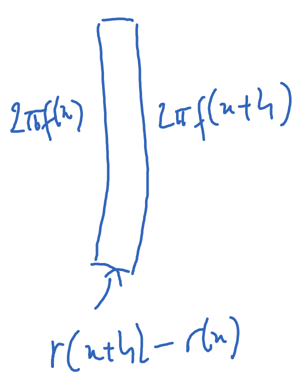
$$S(b) = S(b) - S(a) = \int_a^b S'(x)\,dx$$

$$S'(x) = \lim_{h \to 0} \frac{S(x+h) - S(x)}{h}$$



$S(x+h) - S(x)$ = area of a "ribbon".

Cut this ribbon and straighten it:



area $\approx 2\pi f(x)\left(r(x+h) - r(x)\right)$

Thus,

$$\frac{S(x+h) - S(x)}{h} \approx 2\pi f(x)\,\frac{r(x+h) - r(x)}{h}$$

Let $h \to 0$:

$$S'(x) = 2\pi f(x)\, r'(x) = 2\pi f(x)\sqrt{1 + f'(x)^2}$$

Therefore,

$$\text{area} = S(b) = \int_a^b S'(x)\,dx = \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$$

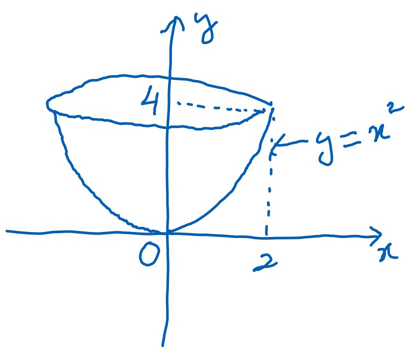
<u>Ex</u> Find the surface area of the region obtained by rotating the curve $y = x^3$ about the x-axis from $x = 1$ to $x = 3$.

$$S = \int_1^3 2\pi y\sqrt{1+y'^2}\,dy$$

$$= \int_1^3 2\pi x^3\sqrt{1+9x^4}\,dx$$

Use the substitution $u = 1+9x^4$, the above integral will be simpler.

<u>Ex</u>



Find area of the surface obtained by rotating the parabola $y = x^2$, $0 \le x \le 2$, about the y-axis.

$$S = \int_0^4 2\pi f(y)\sqrt{1+f'(y)^2}\,dy$$

where $f(y) = \sqrt{y}$.

$$f'(y) = \frac{1}{2\sqrt{y}} \rightsquigarrow f'(y)^2 = \frac{1}{4y}$$

$$S = \int_0^4 2\pi\sqrt{y}\sqrt{1+\frac{1}{4y}}\,dy = \int_0^4 2\pi\sqrt{y+\frac{1}{4}}\,dy = 2\pi\,\frac{2}{3}\left(y+\frac{1}{4}\right)^{3/2}\Big|_0^4$$

$$= \frac{4\pi}{3}\cdot\frac{17^{3/2}-1}{4^{3/2}}$$