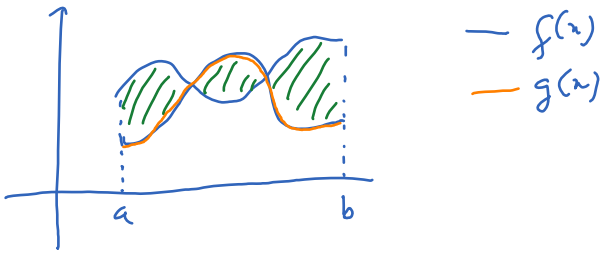
Lecture 3

Friday, September 1, 2023 12:03 AM

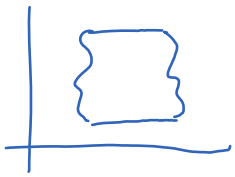
* Prayer

* Area between two curves $y = f(x)$, $y = g(x)$, $x \in [a, b]$ is given by

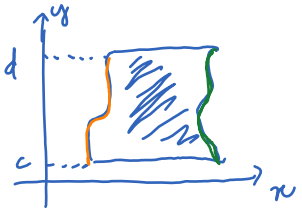
$$\int_a^b |f(x) - g(x)|\, dx$$



Such an area has curvy upper and lower boundaries, but flat left and right boundaries.

* How about areas of the form  ?

Such an area has curvy left and right boundaries, but flat upper and lower boundaries. The solution is to turn the region 90° (thus swapping x and y).

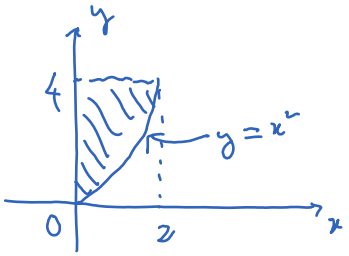


$x = g(y)$

$x = f(y)$

$$\text{Area} = \int_c^d (g(y) - f(y))\, dy$$

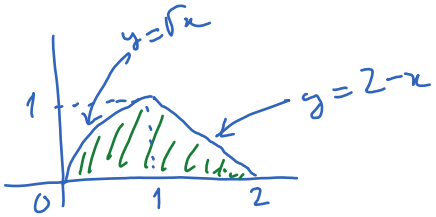
Ex



$$\text{area} = \int_0^2 (4-x^2)\,dx = \int_0^4 (\sqrt{x}-0)\,dx = \frac{16}{3}$$

This area can be computed in two ways.

Ex The second problem on worksheet 8/31.

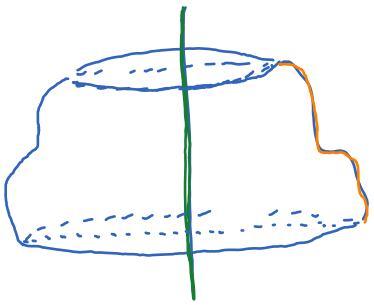


$$\text{area} = \int_0^1 (2-y-y^2)\,dy$$

$$= \int_0^1 (\sqrt{x}-0)\,dx + \int_1^2 (2-x-0)\,dx$$

$$= \frac{7}{6}$$

Solid of revolution



Solid of revolution is a solid obtained by revolving a curve about a fixed axis.

Example: pottery, cone

How to find volume of such a solid?

We cut the solid into thin circular disks,

Then approximate the volume of each disk by the volume of a thin cylinder. Then sum up.

volume of cylinder = $\pi R^2 h$

(πR^2: base area; h: height)