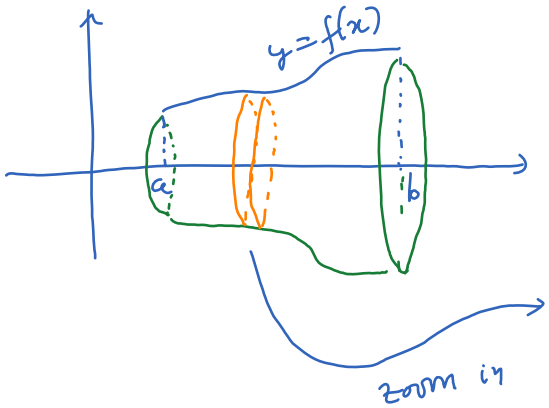
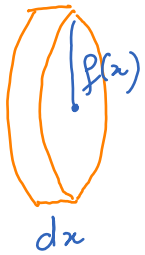
Lecture 4

Tuesday, September 5, 2023 11:22 AM

* Prayer

Volume of solid of revolution



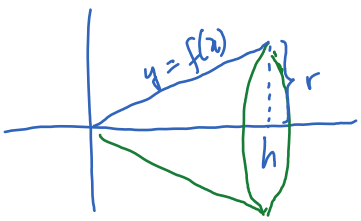


volume of each slice $\approx \pi f(x)^2 dx$

"Sum" of all these slices gives

$$\text{vol} = \int_a^b \pi f(x)^2 dx$$

Ex Find the volume of the cone with base radius r and height h.

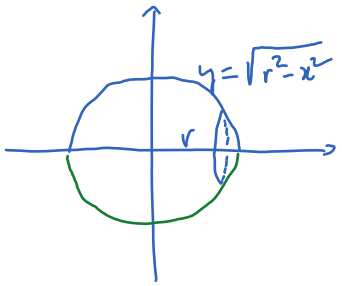


$$\frac{y}{x} = \frac{r}{h} \rightsquigarrow y = \frac{r}{h}x$$

$$\text{vol} = \int_0^h \pi y^2 dx = \int_0^h \pi \frac{r^2}{h^2} x^2 dx$$

$$= \pi \frac{r^2}{h^2} \frac{x^3}{3}\Big|_0^h = \frac{\pi r^2 h}{3}.$$

Ex Find the volume of a sphere with radius r.

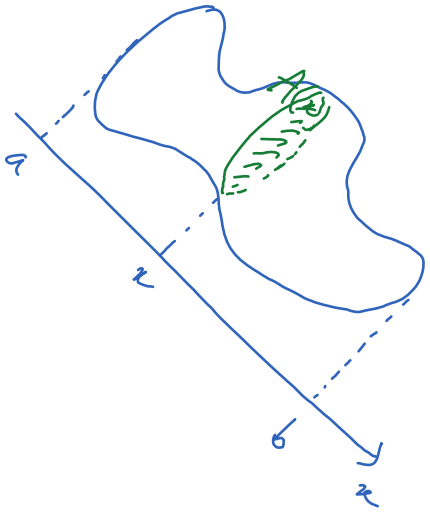


$$vol = \int_{-r}^{r} \pi y^2 dx = \int_{-r}^{r} \pi (r^2 - x^2) dx$$

$$= \pi \left(r^2 x - \frac{x^3}{3} \right) \Big|_{-r}^{r}$$

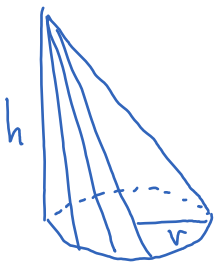
$$= 2\pi \left(r^3 - \frac{r^3}{3} \right) = \frac{4\pi}{3} r^3$$

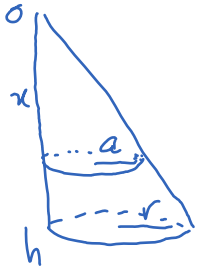
In general, we can compute the volume of a solid if we know the general formula of cross section areas along an axis.



$$vol = \int_a^b A(x) dx$$

Ex Find the volume of the following cone.





$$\frac{x}{h} = \frac{a}{r} \rightsquigarrow a = \frac{r}{h} x$$

$$A(x) = \pi a^2 = \pi \frac{r^2}{h^2} x^2$$

$$vol = \int_0^h A(x) dx = \int_0^h \pi \frac{r^2}{h^2} x^2 dx = \frac{\pi r^2 h}{3}.$$

Using Mathematica:

- On cloud
- on local computer

Some demonstrations in class. How to make a report on Mathematica.