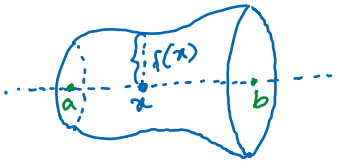
Lecture 5

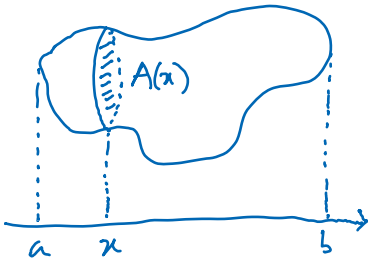
Wednesday, September 6, 2023 12:00 PM

* Prayer



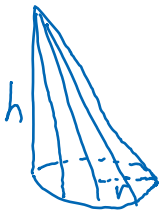
$$vol = \int_a^b \pi f(x)^2 dx$$

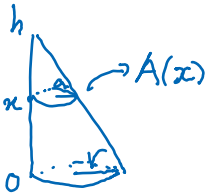
This is a special application of the cross-section method.



$$vol = \int_a^b A(x)dx$$

Ex





Similar triangle :

$$\frac{h-x}{h} = \frac{2a}{2r} = \frac{a}{r}$$

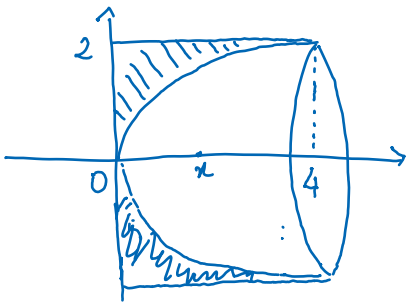
$$\rightsquigarrow \quad a = \frac{r(h-x)}{h}$$

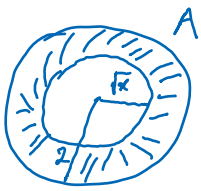
$$A(x) = \pi a^2 = \frac{\pi r^2}{h^2}(h-x)^2$$

$$vol = \int_0^h A(x)dx = \frac{\pi r^2}{h^2}\int_0^h (h-x)^2 dx = \frac{\pi r^2}{h^2}\int_0^h (h^2 - 2hx + x^2)dx$$

$$= \frac{\pi r^2}{h^2}\left(h^2 x - hx^2 + \frac{x^3}{3}\right)\Big|_0^h = \frac{\pi h r^2}{3}$$

Ex

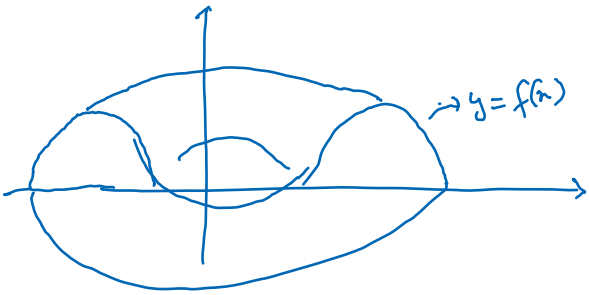




$A(x) = \pi 2^2 - \pi \sqrt{x}^2 = \pi(4-x)$

$$\text{Vol} = \int_0^4 A(x)\,dx = \int_0^4 \pi(4-x)\,dx = \pi\left(4x - \frac{x^2}{2}\right)\Big|_0^4 = \pi\left(4\times 4 - \frac{4^2}{2}\right) = 8\pi$$

Shell method (cylindrical sheel method)



Idea: cut the solid by a cylindrical "knife" instead of a flat knife.