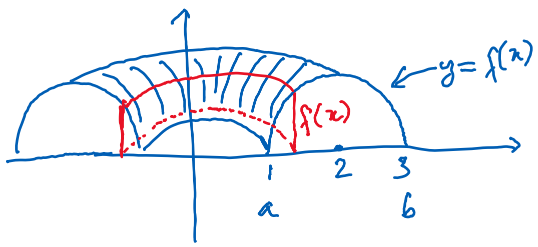
Lecture 6

Wednesday, September 6, 2023 12:55 PM

* Prayer



Partition the solid into shells.

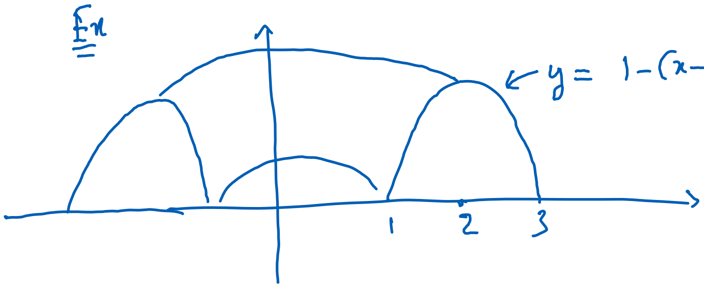
Shell has width dx, height $f(x)$,

inner radius x, outer radius $x + dx$.

The vol of such a shell is $2\pi[(x+dx)^2 - x^2] f(x) \approx 2\pi x f(x) dx$

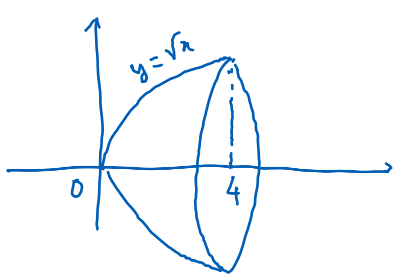
Therefore, the total volume is $\displaystyle\int_a^b \underbrace{2\pi x}_{\text{perimeter}} \underbrace{f(x)}_{\text{height}} \underbrace{dx}_{\text{width}}$.

Ex



$$\text{vol} = \int_1^3 2\pi x [1-(x-2)]^2 dx$$

Ex



$$\text{vol} = \int_0^4 \pi x \, dx \quad \text{(cross sectional method)}$$

$$= \int_0^2 2\pi y(4-y^2)\, dy \quad \text{(shell method)}$$

$$= 8\pi$$

Integration techniques
- substitution
- integration by parts

Substitution:

$$\int f(u)\,u'\,dx = \int f(u)\,du$$

Ex
$$\int_0^1 \sqrt{x^2+1}\;x\,dx = \int_1^2 \sqrt{u}\;\frac{du}{2} = \frac{1}{2}\int_1^2 u^{1/2}du = \frac{1}{2}\,\frac{2}{3}\,u^{3/2}\Big|_1^2$$
$$= \frac{1}{3}\left(2^{3/2}-1\right)$$

$u = x^2+1$

$du = 2x\,dx$

x	0	1
u	1	2

Integration by parts

$$(uv)' = uv' + u'v$$

$$uv' = (uv)' - u'v$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

changing the integrand,
hoping to get an easier problem

Ex
$$\int x e^x\,dx = ?$$

$u = x \rightsquigarrow u' = 1$

$v' = e^x \rightsquigarrow v = e^x$

$$\int x e^x dx = \int uv'\,dx = uv - \int u'v\,dx = xe^x - \int e^x dx = xe^x - e^x + C.$$