Lecture 7

Friday, September 8, 2023 12:38 PM

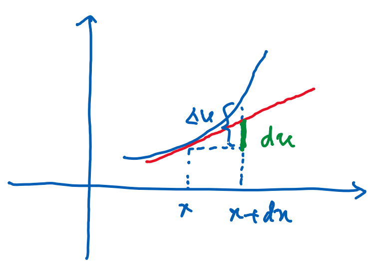
* Prayer

* Two types of integrals $\begin{cases} \text{indefinite integral} \\ \text{definite integral} \end{cases}$ related to each other by the fundament theorem of Calculus.

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$$

* Integration techniques: (work for both types of integral)
  - substitution method: based on the identity $du = u'dx$
  - integration by parts: based on the identity $(uv)' = u'v + uv'$

$$\int \underbrace{f(u)\,u'}_{\text{old integrand}}\,dx = \int \underbrace{f(u)}_{\text{new integrand}}\,du$$



Δu = change in u caused by change in x

du = approximation of Δu that is easier to compute

Ex

$$\int_0^1 x\sqrt{1+x^2}\,dx = \int_?^? x\sqrt{u}\,\frac{du}{2x} = \int_?^? \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int_1^2 u^{1/2}\,du$$

$u = 1 + x^2$

$du = u'dx = 2x\,dx$

$dx = \dfrac{du}{2x}$

Update bounds:

x	0	1
u	1	2

$$= \frac{1}{2}\,\frac{u^{3/2}}{3/2}\Big|_1^2$$

$$= \frac{1}{3}\left(2^{3/2} - 1\right)$$

Work on the second problem of Worksheet 9/6/2023.

$$\int_0^1 x^2\sqrt{1-x^3}\,dx = \int_1^0 x^2\sqrt{u}\left(-\frac{du}{3x^2}\right) = -\frac{1}{3}\int_1^0 \sqrt{u}\,du$$

$$= \frac{1}{3}\int_0^1 u^{1/2}\,du = \frac{1}{3}\left.\frac{u^{3/2}}{3/2}\right|_0^1 = \frac{2}{9}$$

$u = 1-x^3$

$du = u'dx = -3x^2dx$

$dx = -\frac{du}{3x^2}$

x	0	1
u	1	0