Lecture 5

Monday, September 11, 2023 5:26 PM

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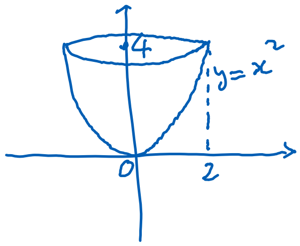
Quiz:



$$\text{area} = \int_0^2 (4-x^2)\,dx = 4x - \frac{x^3}{3}\Big|_0^2 = 8 - \frac{8}{3} = \frac{16}{3}.$$

$$\text{vol} = \int_0^4 \pi y\, dy = 8\pi \quad \text{(using cross section)}$$

$$= \int_0^2 2\pi x(4-x^2)\,dx \quad \text{(using shell method)}$$

$$= 8\pi$$

For extra credit, find the volume using the cross section method with the axis being the x-axis instead of the y-axis.

<u>Integration by parts</u>

$$(uv)' = u'v + uv'$$

$$\rightsquigarrow \quad u'v = (uv)' - uv'$$

$$\rightsquigarrow \quad \underbrace{\int u'v\,dx}_{\text{old problem}} = uv - \underbrace{\int uv'\,dx}_{\text{new problem}}$$

Ex $\quad \int x \sin x\, dx$

There are many ways to choose u and v:

(A) $\begin{cases} u' = x \\ v = \sin x \end{cases}$ (B) $\begin{cases} u' = \sin x \\ v = x \end{cases}$ (C) $\begin{cases} u' = x\sin x \\ v = 1 \end{cases}$ (D) $\begin{cases} u' = 1 \\ v = x\sin x \end{cases}$

Try (A): $\quad u = \frac{x^2}{2}$

$$\int \underbrace{x \sin x}_{u'v} dx = uv - \int uv' dx = \frac{x^2}{2}\sin x - \underbrace{\int \frac{x^2}{2}\cos x dx}$$

difficult, even more than the original problem

Try (B):

$$\begin{cases} u' = \sin x \\ v = x \end{cases} \rightsquigarrow \begin{cases} u = -\cos x \\ v' = 1 \end{cases}$$

$$\int \underbrace{x \sin x}_{u'v} dx = uv - \int uv' dx = -x\cos x - \int -\cos x dx = -x\cos x + \int \cos x dx$$

$$= -x\cos x + \sin x + C$$

Sometimes, we need to combine two methods: substitution and integration by parts.

Ex $\displaystyle\int_0^4 e^{\sqrt{x}} dx$

$u = \sqrt{x}$

$du = u' dx = \dfrac{1}{2\sqrt{x}} dx = \dfrac{1}{2u} dx$

$dx = 2u du$

$$\int_0^4 e^{\sqrt{x}} dx = \int_0^2 e^u \, 2u du$$

We will continue next time.