Definitions

- A **proposition** makes a claim (either an assertion or a denial) that may be either true or false. It must have the structure of a complete sentence.
- Any proposition has two possible **truth values**: T = true or F = false.
- A **truth table** is a table with a row for each possible set of truth values for the propositions being considered.

Example

- State whether or not the following are propositions:

- There is a tornado.
- Yes

- Can you give me a hand with this?
- No

Negation (Opposites)

The **negation** of a proposition *p* is another proposition that makes the opposite claim of *p*.

p	*not p*
T	F
F	T

← If p is true (T), *not p* is false (F).

← If p is false (F), *not p* is true (T).

Symbol: ~

Logical Connectors

Propositions are often joined with **logical connectors**—words such as *and*, *or*, and *if…then*.

Example:

p = I won the game.
q = It was fun.

Logical Connector	**New Proposition**
and	I won the game and it was fun.
or	I won the game or it was fun.
if…then	If I won the game, then it was fun.

And Statements (Conjunctions)

Given two propositions *p* and *q*, the statement *p and q* is called their **conjunction**. It is true only if *p* and *q* are both true.

p	*q*	*p and q*
T	T	T
T	F	F
F	T	F
F	F	F

Symbol: $\wedge$

Example

- State each proposition in the following statement and their truth values. Then state whether the statement is true.

- Some people are awake and some people are pale.

- Some people are awake: True
- Some people are pale: True
- The statement is true.

Or Statements (Disjunctions)

Given two propositions *p* and *q*, the statement *p or q* is called their **disjunction**. It is true unless *p* and *q* are both false.

p	*q*	*p or q*
T	T	T
T	F	T
F	T	T
F	F	F

Symbol: $\vee$

Or Statements (Disjunctions)

The word ***or*** can be interpreted in two distinct ways:

- An **inclusive *or*** means "either or both."
- An **exclusive *or*** means "one or the other, but not both."

In logic, assume *or* is inclusive unless told otherwise.

Example

- State each proposition in the following statement and their truth values. Then state whether the statement is true.

- $10 \times 8 = 80 \ or\ 3 + 8 = 10$

- $10 \times 8 = 80$: True
- $3 + 8 = 10$: False
- The statement is true.

If… Then Statements (Conditionals)

A statement of the form *if p, then q* is called a **conditional proposition** (or implication). It is true unless *p* is true and *q* is false.

p	q	*if p, then q*
T	T	T
T	F	F
F	T	T
F	F	T

- Proposition *p* is called the **hypothesis**.
- Proposition *q* is called the **conclusion**.

Example

- State the hypothesis and conclusion of the following conditional statement and their truth values. Then, state whether the following propositions are true.

- If eagles can fly, then eagles are fish.
- Hypothesis: Eagles can fly: True
- Conclusion: Eagles are fish: False
- The statement is false.

Example

- State the hypothesis and conclusion of the following conditional statement and their truth values. Then, state whether the following propositions are true.

- If 13+7=22, then America has a president.
- Hypothesis: 13+7=22: False
- Conclusion: America has a president. True
- The statement is true.

Alternative Phrasings of Conditionals

The following are common alternative ways of stating *if p, then q*:

- *p* is sufficient for *q*
- *p* will lead to *q*
- *p* implies *q*
- *q* is necessary for *p*
- *q* if *p*
- *q* whenever *p*

Variations on the Conditional

Conditional:

If *p*, then *q*

If it is raining, then I will bring an umbrella to work.

Converse:

If *q*, then *p*

If I bring an umbrella to work, then it must be raining.

Inverse:

If ~*p*, then ~*q*

If it is not raining, then I will not bring an umbrella to work.

Contrapositive:

If ~*q*, then ~*p*

If I do not bring an umbrella to work, then it must not be raining.

Logical Equivalence

Two statements are **logically equivalent** if they share the same truth values.

logically equivalent: *if p, then q* and *if ~q, then ~p* (contrapositive)

p	q	$\sim p$	$\sim q$	*if p, then q*	*if q, then p* (converse)	*if ~p, then ~q* (inverse)	*if ~q, then ~p* (contrapositive)
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

logically equivalent: *if q, then p* (converse) and *if ~p, then ~q* (inverse)