

# Lecture 10

Tuesday, September 24, 2024 1:07 PM

Saving vs loan: saving is an inflow of cash while loan is an outflow of cash.

Saving: the interest rate works for you

Loan: the interest rate works against you

A stream of fixed periodic payment for saving is called an **annuity**.

A stream of fixed periodic payment to pay off a loan is called an **amortization**.

Consider a stream of fixed periodic payment of  $q$  dollars over  $N$  periods. The interest rate per period is  $i$ . If this stream of payments is for saving, then you are interested in knowing the **future value** of it, which is

$$A = q \frac{(1+i)^N - 1}{i}$$

This is how much you will get after  $N$  periods.

On the other hand, if the stream of payments is for paying off the loan, you are interested in knowing your original debt and how much you have left to pay. In other words, you are interested in the **present value** of the loan.

You are currently at the beginning of month 1. Suppose you make the payment at the end of each month.

At the end of month 1: the present value of payment is  $\frac{q}{1+i}$

At the end of month 2: the present value of payment is  $\frac{q}{(1+i)^2}$

At the end of month 3: the present value of payment is  $\frac{q}{(1+i)^3}$

...

At the end of month  $N$ : the present value of payment is  $\frac{q}{(1+i)^N}$

In total, at the present, the whole payment stream is worth

$$\begin{aligned} P &= \frac{q}{1+i} + \frac{q}{(1+i)^2} + \dots + \frac{q}{(1+i)^N} = \frac{q}{1+i} \left( 1 + \frac{1}{1+i} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^{N-1}} \right) \\ &= \frac{q}{1+i} \frac{1 - \frac{1}{(1+i)^N} - 1}{\frac{1}{1+i} - 1} \\ &= \frac{q}{i} \left( 1 - \frac{1}{(1+i)^N} \right) \end{aligned}$$

Therefore, the **present value** of the payment stream is

$$P = \frac{q}{i} \left( 1 - \frac{1}{(1+i)^N} \right)$$

In other words, you have to pay  $q$  dollars each period for  $N$  periods to payoff the loan of  $P$  dollars taken at the present.

*Example:* you want to buy a house and can afford to pay \$2000/month. The APR is 5%. The loan term is 30 years. What is the maximum price of the house that you can afford? How much of your monthly payment go toward principal, and how much of it go to pay the interest at the end of month 1, at the end of month 2, and at the end of month 3?

*Example:* Now answer the same question but assume that you have saving to put down 20% of the total price of the house.