Lecture 9

Friday, September 20, 2024 9:09 AM

Recall the continuous compounding:

$$A = Pe^{n*APR}$$

where n is the number of years.

For example: you put $1000 in the bank with interest rate $APR = 1\%$ compounded continuously. What will you have in 30 months?

30 months = $\frac{30}{12} = 2.5$ years.

You will have $A = 1000e^{1.5*0.01} \approx 1015$

Total return = $\frac{new\ value - current\ value}{current\ value}$

APY = total return in 1 year

Example: you put in the bank $1000. In 5 years, your money will be $1020. What is the total return?

Total return = $\frac{1020-1000}{1000} = \frac{20}{1000} = 2\%$

Saving plans: annuity vs lump sum

Annuity = financial plan that requires periodic and fixed payments over a course of time

Lump sum = financial plan that requires only one-time payment

Example: you start with $0 in your bank account and put $50 in the bank account at the end of each month. The bank pays an interest rate of 0.5% per month.

At the end of month 1, you have 50 (no interest generated yet)

At the end of month 2, you have 50 + 50*(1+0.005)

At the end of month 3, you have 50 + 50*(1+0.005) + 50*$(1+0.005)^2$

...

At the end of month N, you have 50 + 50*(1+0.005) + 50*$(1+0.005)^{N-1}$

With some algebra, one can write

50 + 50*(1+0.005) + 50*$(1+0.005)^{N-1}$=50$(1 + p + p^2 + \cdots + p^{N-1})$=50$\frac{p^N-1}{p-1}$=50 $\frac{(1+0.005)^N-1}{0.005}$

In general, the balance at the end of the N'th compounding period is

$$A = p\frac{(1+i)^N - 1}{i}$$

Where p is the fixed periodic payment, i is the interest rate per period, and N is the number of periods.

Example: in the example above, compare your balance with the annuity plan with fixed payment $50/month with APR=3% with the balance with the lump sum plan, where you make a one-time deposit of $4000 at APR=2%, in 5, 10, 20 years.