Lab 1

In this lab, you will learn how to:

- do basic arithmetic on numbers,
- define functions and graph them,
- find limits of a function.

The next lab will be built upon this lab, so please make sure that you go through all the instruction carefully and do all the assignments.

1 To turn in

Do Problems 1-35 in a single Mathematica Notebook file, or ipynb file if you use JupyterLab. Write *your name* and *lab number* at the beginning of your report. Clearly label each problem to separate them from other problems. Make sure to comment on each problem. If your code doesn't run correctly, explain what you are trying to do. **Failed code without any comment/explanation will receive 0 point.** Submit on Canvas both the *pdf file* and the *source file* (nb or ipynb).

Problems	Points
1-3, 6, 7, 9-13, 16-18, 20-22, 25, 26, 30	1
4, 5, 8, 14, 15, 19, 23, 24, 27-29	2
31-35	3
Readability of your report	3
Total: 35	Total: 59

2 Basic arithmetic

(1) Type `(1+2)^2-3*a` followed by `Shift+Enter`.

(2) Type `a=5` followed by `Enter`. Then type `(1+2)^2-3*a` followed by `Shift+Enter`.

(3) Type `35/6`, then `Shift+Enter`. Do it give you a fraction or a decimal-point number?

(4) Type `(1+2)^2` followed by `Shift+Enter`. Then type `[1+2]^2` followed by `Shift+Enter`. What can you learn from this experiment?

(5) Type `N[35/6]` followed by `Shift+Enter`. Then type `N[35/6,10]` followed by `Shift+Enter`. Then type `N[35/6,20]` followed by `Shift+Enter`. What does the command `N[...]` do?

(6) The number $\sqrt{2}$ in Mathematica is `Sqrt[2]` (notice the capitalized S). Type `Sqrt[2]` followed by `Shift+Enter`. Does it give you a decimal-point number?

(7) Type `N[Sqrt[2]]` followed by `Shift+Enter`. An equivalent way of writing this is `Sqrt[2] // N`

(8) Find $\sqrt{2}$ using 10 significant digits.

(9) Type `Sin[Pi]` followed by `Shift+Enter`. Next, try the same command but with lowercase S and/or P. Is Mathematica case sensitive?

(10) Type `47^100;` (with the semicolon) followed by `Shift+Enter`. Then type `47^100` (without semicolon) followed by `Shift+Enter`.

The semicolon is to hold the output. One uses it when output is too long or not of interest. You may have noticed that the function `N` is to evaluate a numerical value of an expression. Each function's name is capitalized is followed by a pair of square brackets `[...]` instead of parentheses (...) as we normally write on paper. For example, the function sin(x), cos(x), tan(x), exp(x), ln(x) are written as `Sin[x]`, `Cos[x]`, `Tan[x]`, `Exp[x]`, `Log[x]` respectively in Mathematica.

(11) The number e is typed as `Exp[1]` or `E`. Find the value of e using 10 significant digits.

(12) Find the value of $\ln 2$ using 10 significant digits.

(13) Use a suitable property of logarithm to find $\log_2 3$ using 10 significant digits.

(14) Find a numerical value of $e^{2(\cos(\sqrt{2})+\sin(5))} + \ln 2$. Be careful to use square brackets and parentheses properly.

3 Define and graph functions

To define a function, we use a combination of the underscore sign "_" and the assignment operator ":=" as follows.

(15) `f[x_] := Sin[x]+Cos[x]` (notice the underscore after `x`) followed by `Shift+Enter`. Then type `f[Pi]+f[Pi/4]` followed by `Shift+Enter`. Can you find a numerical value of this number?

(16) Repeat the previous exercise but now replace the name `f` by a different name (say `g`) and drop the underscore after x. What do you learn about the role of the underscore sign?

Note that the underscore is required only when *defining* a function. It is not needed when *calling* the function.

(17) Define two functions $f(x) = \ln x$ and $g(x) = \tan(\cos x)$. Then evaluate numerically the quantities $f(f(10))$ and $f\left(1 + g\left(\frac{\pi}{8}\right)\right)$ with 4 significant digits.

(18) To clear a quantity from the memory, use the command `Clear`. Type `Clear[f]` followed by `Shift+Enter`. Then type `f[Pi]+f[Pi/4]` followed by `Shift+Enter`. What do you observe?

(19) Define the function $f(x) = \sin(xe^x)$ and then evaluate $f(e^2)$ with 8 significant digits.
Warning: you can type the product xe^x as either `x*E^x` or `x E^x` (space between `x` and `E^x`). If you type `xE^x`, Mathematica will understand `xE` as the name of a single variable.

Now let us plot functions of one variable. Try the following command:

(20) `Plot[Sin[x]+Cos[2x], {x,0,2*Pi}]`, then `Shift+Enter`.

(21) To customize the style of the graph, you can add the option `PlotStyle` to the `Plot` command. Try the following:

```
Plot[Sin[x]+Cos[2x], {x,0,2*Pi}, PlotStyle -> {Red, Dashed}]
```

Then `Shift+Enter`. Note that the arrow is typed as $- >$.

(22) You can also give the function a name before plotting it. For example,

```
f[x_] := Sin[x]+Cos[2x];
Plot[f[x], {x,0,2*Pi}, Filling -> Axis]
```

Then `Shift+Enter`. In the second command, note that the underscore within the brackets is no longer needed because f was already defined.

(23) To graph two functions, say sin(x) and cos(x), on the same plot, we do the following:

```
Plot[{Sin[x],Cos[x]},{x,0,5*Pi}]
```

To graph sin(x) with dashed line with green color, and graph cos(x) with thin line with red color, do the following:

```
Plot[{Sin[x],Cos[x]},{x,0,5*Pi}, PlotStyle->{{Dashed,Green},{Thin,Red}}]
```

To label the graphs, add the option `PlotLegends->Automatic` into the `Plot` command.

(24) Graph the functions x, x^2, x^3, x^4, x^5 on the interval $x \in [0, 1]$ on the same plot. Describe the behavior of x^n as n increases.

A piecewise function is a function given by different formulas for each given interval. For example,

$$f(x) = \begin{cases} 2x & \text{if} \quad x \geq 0 \\ x^2 & \text{if} \quad x < 0 \end{cases}, \qquad g(x) = \begin{cases} \sin x & \text{if} \quad x < -1 \\ e^x & \text{if} \quad -1 \leq x \leq 1 \\ \cos x & \text{if} \quad x > 1 \end{cases}$$

are piecewise functions. We use the command **Piecewise** to define a piecewise function. The syntax is as follows:

`Piecewise[{{`val_1`,`$cond_1$`},{`val_2`,`$cond_2$`},...}]`

represents a piecewise function with values val_i in the regions defined by the conditions $cond_i$.

(25) Use the following commands to define the function $f(x)$ above and plot it.

```
f[x_] := Piecewise[{{2 x, x >= 0}, {x^2, x < 0}}]
Plot[f[x], {x, -2, 2}]
```

Make sure that you don't miss the underscore after x. Press `Enter` to go from the first line to the second line. Then press `Shift+Enter` to execute the entire block.

4 Find limits of a function

To compute the limit

$$\lim_{x \to 1} \frac{x^2 + \sin(\pi x)}{x + 1}$$

the first thing to try is to plug $x = 1$ into the function. Then you see that the answer is 1/2.

(26) Now let us verify this answer experimentally. We will make a table of values of $f(x)$ as x gets closer and closer to 1. First, we define the function f:

```
f[x_] := (x^2 + Sin[Pi*x])/(x+1)
```

Then we make a table of values of $f(x)$ as x increases from 0.8 to 0.99 by an increment of 0.01.

```
Table[{x,f[x]}, {x,0.8,0.99,0.01}] // TableForm
```

(27) Make a table of values of $f(x)$ as x increases from 0.99 to 0.999 by an increment of 0.001. Does it look like $f(x)$ approaches 1/2?

(28) Make a table of values of $f(x)$ as x decreases from 1.01 down to 1.0001 by an increment of -0.0001. Does it look like $f(x)$ approaches 1/2?

(29) Another way to verify $\lim_{x\to 1} f(x) = 1/2$ is by graphing the function f near $x = 1$. Graph the function f on the interval $x \in [0, 2]$ to see if $\lim_{x\to 1} f(x) = 1/2$.

(30) You can also compute the limit using the command **Limit**. Enter the following commands in the same input block. Press `Enter` to go from one line to the next, and press `Shift+Enter` to execute the entire block:

```
Limit[f[x], x->1]
Limit[f[x], x -> 1, Direction -> "FromAbove"]
Limit[f[x], x -> 1, Direction -> "FromBelow"]
```

(31) Evaluate the one-sided limits

$$\lim_{x\to 0^-} \frac{\sin x}{|x|}, \qquad \lim_{x\to 0^+} \frac{\sin x}{|x|}$$

using three different methods: 1) making a table of values, 2) graphing, and 3) using the command **Limit**. Does the limit $\lim_{x\to 0} \frac{\sin x}{|x|}$ exist? Note that $|x|$ is typed as `Abs[x]`.

(32) Evaluate the limits

$$\lim_{x\to\infty} \frac{3x-2}{\sqrt{4x^2+1}}, \qquad \lim_{x\to-\infty} \frac{3x-2}{\sqrt{4x^2+1}}$$

using three different methods: 1) making a table of values, 2) graphing, and 3) using the command **Limit**. Note that the infinity symbol ∞ is typed as `Infinity`.

(33) Define the piecewise function

$$g(x) = \begin{cases} \sin x & \text{if} \quad x < -1 \\ e^x & \text{if} \quad -1 \le x \le 1 \\ \cos x & \text{if} \quad x > 1 \end{cases}$$

Plot the function on the interval $[-2, 2]$. Then evaluate numerically (using 6 significant digits) the one-side limits of g as $x \to -1^-$, $x \to -1^+$, $x \to 1^-$, $x \to 1^+$.

(34) Define and plot the function $f(x) = \sin\left(\frac{1}{x}\right)$ on the interval $[-1, 1]$. Then plot f on the following intervals: $[-0.1, 0.1]$, $[-0.01, 0.01]$, and $[-0.001, 0.001]$. The purpose of doing so is to "zoom in" the graph of f near $x = 0$. Does it look like f has a limit as $x \to 0$? Explain why. Double check your answer using the command **Limit**.

(35) Define the piecewise function

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if} \quad x \ne 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

In the **Piecewise** command, the unequal sign $\ne$ is typed as `!=` and the equal sign $=$ is typed as `==`. Plot the function on the interval $[-1, 1]$. Then evaluate the limit of f as $x \to 0$. Is this function continuous at $x = 0$?