Lecture 13

Saturday, September 21, 2024 2:30 AM

Some applications of Intermediate Value Theorem: show that an equation has a solution.

The IVT has a variation that is more commonly used:
If f is continuous on $[a, b]$ and M lies between $f(a)$ and $f(b)$ then there exists $c \in [a, b]$ such that $f(c) = M$.
Graphic interpretation…

Example: show that there is a solution to the equation x + sin x = 4
It would be extremely difficult to find this solution exactly. But that is not what we are asked to do. Let $f(x) = x + \sin x$.
Then f is continuous on $R = (-\infty, \infty)$. Moreover, $f(0) = 0 < 4$ and $f(6) = 6 + \sin 6 > 4$. There must be a number $c \in (4,6)$ such that $f(c) = 4$.

Bisection method: cutting the search interval by half each time.

Vertical and horizontal asymptotes:

$$\lim_{x\to\infty} \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0} = \lim_{x\to\infty} \frac{a_n x^n}{b_m x^m}$$

(only the highest powers matter).

$$f(x) = \frac{8 - x^3}{1 - x - x^2 + x^3}$$

$$f(x) = \sqrt{x^2 + 4} - x$$