

Lecture 2

Thursday, September 5, 2024 12:19 AM

Catalog of elementary functions:

polynomials : $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

rational : $P(x)/Q(x)$

exponential : a^x

logarithm : $\log_a x$

trigonometry : $\sin x, \cos x, \dots$

power function : x^n

root function : $\sqrt[n]{x}$

Domain of a function f is the set of all x 's on which $f(x)$

is well-defined.

Ex $f(x) = \frac{x}{x-1}$

domain = $\mathbb{R} \setminus \{1\}$

Ex $f(x) = \frac{x^2}{x}$

domain = $\mathbb{R} \setminus \{0\}$, although it is tempting to think

domain = \mathbb{R} because $f(x) = x$.

Ex $f(x) = x^2 + 3x + 2$

domain = \mathbb{R}

Ex $f(x) = \frac{x}{x^2 + 3x + 2}$

domain of $f = \mathbb{R} \setminus \{x \mid x^2 + 3x + 2 = 0\}$

$x^2 + 3x + 2 = (x+1)(x+2) \rightarrow$ equal to 0 if $x = -1$ or $x = -2$

domain of $f = \mathbb{R} \setminus \{-1, -2\}$.

Ex $f(x) = \sqrt[3]{x-1}$

Domain of $f = \mathbb{R}$

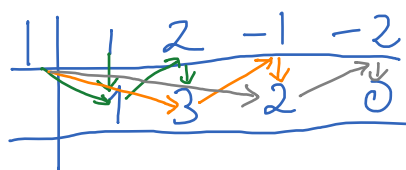
Ex $f(x) = \sqrt{x-1}$

Domain of $f = \{x \mid x-1 \geq 0\} = \{x \mid x \geq 1\} = [1, \infty)$

Ex $f(x) = \sqrt{x^3 + 2x^2 - x - 2}$

domain of $f = \{x \mid x^3 + 2x^2 - x - 2 \geq 0\}$

We need to factor $x^3 + 2x^2 - x - 2$.



$$\begin{aligned} x^3 + 2x^2 - x - 2 &= (x-1)(x^2 + 3x + 2) \\ &= (x-1)(x+1)(x+2) \end{aligned}$$

x	-2	-1	1	
$x-1$	$-$	$-$	$-$	$+$
$x+1$	$-$	$-$	$+$	$+$
$x+2$	$-$	$+$	$+$	$+$
product	$-$	$+$	$-$	$+$

$(x-1)(x+1)(x+2) \geq 0$ if and only if $x \in \underbrace{[-2, -1] \cup [1, \infty)}_{\text{domain of } f}$

Ex $f(x) = \frac{\cos x}{1-2\sin x}$

Domain = $\mathbb{R} \setminus \{x \mid 1-2\sin x = 0\} = \mathbb{R} \setminus \left\{x \mid \sin x = \frac{1}{2}\right\}$

$\sin x = \frac{1}{2} \Leftrightarrow \sin x = \sin \frac{\pi}{6} \rightsquigarrow x = \frac{\pi}{6} + k2\pi$

or $x = \frac{5\pi}{6} + k2\pi$

Domain = $\mathbb{R} \setminus \left\{ \frac{\pi}{6} + k2\pi, \frac{5\pi}{6} + k2\pi \mid k \in \mathbb{Z} \right\}$

* Give an example of a function f such that $f(1) = f(2) = 0$:

$f(x) = (x-1)(x-2)$

* Give an example of a function f s.t. $f(1) = f(2) = 0, f(3) = 1$:

$f(x) = a(x-1)(x-2)$

$f(3) = a(3-1)(3-2) = 2a$

For $f(3) = 1$, we choose $a = \frac{1}{2}$. Hence, $f(x) = \frac{1}{2}(x-1)(x-2)$.