Lecture 2

Thursday, September 5, 2024 12:19 AM

Catalog of elementary functions:

polynomials : $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

rational : $P(x)/Q(x)$

exponential : a^x

logarithm : $\log_a x$

trigonometry : $\sin x, \cos x, \ldots$

power function : x^n

root function : $\sqrt[n]{x}$

<u>Domain</u> of a function f is the set of all x's on which $f(x)$ is well-defined.

<u>Ex</u> $f(x) = \dfrac{x}{x-1}$

domain $= \mathbb{R} \setminus \{1\}$

<u>Ex</u> $f(x) = \dfrac{x^2}{x}$

domain $= \mathbb{R} \setminus \{0\}$, although it is tempting to think

domain $= \mathbb{R}$ because $f(x) = x$.

Ex $f(x) = x^2+3x+2$

domain $= \mathbb{R}$

Ex $f(x) = \dfrac{x}{x^2+3x+2}$

domain of $f = \mathbb{R} \setminus \{x \mid x^2+3x+2=0\}$

$x^2+3x+2 = (x+1)(x+2) \longrightarrow$ equal to 0 if $x=-1$ or $x=-2$

domain of $f = \mathbb{R}\setminus\{-1,-2\}$.

Ex $f(x) = \sqrt[3]{x-1}$

Domain of $f = \mathbb{R}$

Ex $f(x) = \sqrt{x-1}$

Domain of $f = \{x \mid x-1 \geq 0\} = \{x \mid x \geq 1\} = [1,\infty)$

Ex $f(x) = \sqrt{x^3+2x^2-x-2}$

domain of $f = \{x \mid x^3+2x^2-x-2 \geq 0\}$

We need to factor x^3+2x^2-x-2.

$$\begin{array}{c|cccc} 1 & 1 & 2 & -1 & -2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$x^3+2x^2-x-2 = (x-1)(x^2+3x+2)$

$= (x-1)(x+1)(x+2)$

x		-2		-1		1	
$x-1$	$-$	|	$-$	|	$-$	0	$+$
$x+1$	$-$	|	$-$	0	$+$	|	$+$
$x+2$	$-$	0	$+$	|	$+$	|	$+$
product	$-$	0	$+$	0	$-$	0	$+$

$(x-1)(x+1)(x+2) \geq 0$ if and only if $x \in \underbrace{[-2,-1] \cup [1,\infty)}_{\text{domain of } f}$

<u>Ex</u> $\quad f(x) = \dfrac{\cos x}{1-2\sin x}$

Domain $= \mathbb{R} \setminus \{x \mid 1-2\sin x = 0\} = \mathbb{R} \setminus \{x \mid \sin x = \frac{1}{2}\}$

$\sin x = \frac{1}{2} \iff \sin x = \sin\frac{\pi}{6} \rightsquigarrow x = \frac{\pi}{6} + k2\pi$

or $x = \frac{5\pi}{6} + k2\pi$

Domain $= \mathbb{R} \setminus \{\frac{\pi}{6} + k2\pi, \frac{5\pi}{6} + k2\pi \mid k \in \mathbb{Z}\}$

* Give an example of a function f such that $f(1) = f(2) = 0$:

$f(x) = (x-1)(x-2)$

* Give an example of a function f s.t. $f(1) = f(2) = 0$, $f(3) = 1$:

$f(x) = a(x-1)(x-2)$

$f(3) = a(3-1)(3-2) = 2a$

For $f(3) = 1$, we choose $a = \frac{1}{2}$. Hence, $f(x) = \frac{1}{2}(x-1)(x-2)$.