Lecture 21

Some applications of the identity $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

Ex: find the limit
$$\lim_{x \to 0} \frac{\tan x}{x}$$

Ex: find the limit
$$\lim_{x \to 0} \frac{\sin(2x)}{x}$$

Ex: find the limit
$$\lim_{x \to 0} \frac{\sin(3x) \tan(5x)}{2x^2}$$

Chain rule:
$y = f(g(x))$. What is y' ?

$$[f(g(x))]' = \lim_{x \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$
$$= \lim_{x \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \frac{g(x+h) - g(x)}{h} = f'(g(x))g'(x)$$

Ex: find $(\sin(x^2))'$

Ex: find $(\sin(\cos(2x + 1)))'$

Ex: find the vertical velocity $y'(t)$ knowing the trajectory $y = y(x)$ and the horizontal velocity $x'(t)$.

Some exercises are on the worksheet.