Lecture 22

Wednesday, October 9, 2024 11:55 PM

Chain rule:
$y = f(g(x))$. What is y' ?

$$[f(g(x))]' = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$
$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \frac{g(x+h) - g(x)}{h}$$
$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

Let $u = g(x)$ and $v = g(x+h)$. Note that u is fixed as $h \to 0$ and $v \to u$ as $h \to 0$. Thus,
$$\lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} = \lim_{v \to u} \frac{f(v) - f(u)}{v - u} = f'(u)$$

Therefore,
$$[f(g(x))]' = f'(g(x))g'(x)$$

Ex: find $(\sin(x^2))'$

Ex: find $(\sin(\cos(2x+1)))'$

Ex: find the vertical velocity $y'(t)$ knowing the trajectory $y = y(x)$ and the horizontal velocity $x'(t)$.

Tips:
With $u = u(x)$, we have
$$\frac{d}{dx}[\sin u] = u' \cos u$$
$$\frac{d}{dx}[e^u] = u' e^u$$
$$\frac{d}{dx}[f(u)] = u' f(u)$$