Lecture 5

Monday, September 9, 2024 11:33 PM

* Problem 3 of the worksheet yesterday:

$$f(x) = \frac{x}{(2^{x+1}-1)(3^x - \frac{1}{9})}$$

f is well-defined if $(2^{x+1}-1)(3^x-\frac{1}{9}) \neq 0$, which is equivalent to both $2^{x+1}-1 \neq 0$ and $3^x - \frac{1}{9} \neq 0$.

$2^{x+1}-1 = 0$ if and only if $2^{x+1} = 1 = 2^0 \rightsquigarrow x = -1$

$3^x - \frac{1}{9} = 0$ if and only if $3^x = \frac{1}{3} = 3^{-2} \rightsquigarrow x = -2$

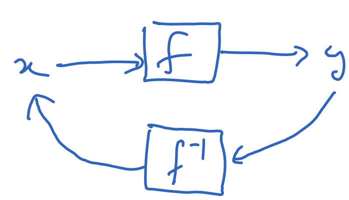
Thus, the domain of f is $\mathbb{R} \setminus \{-1,-2\} = (-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$.

<u>Inverse function</u>

Inverse of $2x$ is $\frac{x}{2}$

Inverse of x^2 $(x \geq 0)$ is $\sqrt{x}$

Inverse of $\frac{1}{x}$ is $\frac{1}{x}$



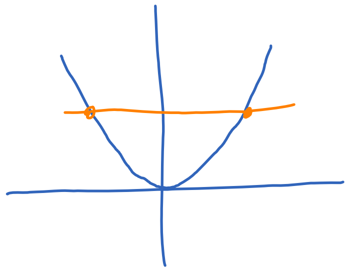
$$3 \xrightarrow{2x} 6 \xrightarrow{\frac{x}{2}} 3$$

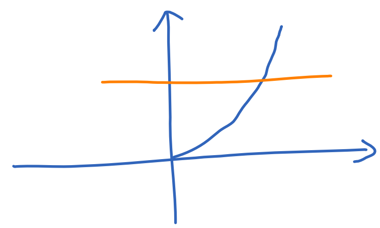
$$3 \xrightarrow{x^2} 9 \xrightarrow{\sqrt{x}} 3$$

$$3 \xrightarrow{\frac{1}{x}} \frac{1}{3} \xrightarrow{\frac{1}{x}} 3$$

One-to-one function: one y comes from only one x

one output comes from only one input.





$f(x)=x^2$ is not one-to-one because it can happen that one y corresponds to two x's.

However, $f(x)=x^2$, $x \geq 0$, is one-to-one.

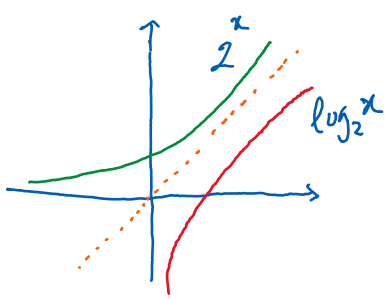
$\star$ To check if a curve is the graph of a function, we use the Vertical Line Test. To check if a function is one-to-one, we use the Horizontal Line Test.

<u>Ex</u> $\quad f(x) = \dfrac{1-x}{2-x}$

Find f^{-1}

<u>Logarithm functions</u>

$f(x) = b^x \rightsquigarrow f^{-1}(x) = \log_b x$



From the perspective of f, x is the input and y is the output.

From the perspective of f^{-1}, y is the input and x is the output.

The graph of f^{-1} is the mirror reflection of the graph of f wrt the line $y=x$.