Lecture 6

Wednesday, September 11, 2024 1:02 AM

Limit is the central notion of Calculus. It is used to define derivatives and integrals, which are two the major concepts of this course.

Consider two quantities x and y, where y depends on x. You can think of y as a function of x. For example, as you are driving, x is time and y is the distance you have travelled up to time x.

Suppose that as x goes to some value, say a, (meaning the value of x is getting closer and closer to a), the value of y gets closer and closer to some value, say b. We will say that the "limit" of y as x goes to a is b. In notation:

$$\lim_{x \to a} y = b$$

For example, consider $y = x^2$. It is conceivable that as x gets closer and closer to 2, y will get closer and closer to $2^2 = 4$. This can be checked by a calculator: try x=1.9, 1.99, 1.999, 2.1, 2.01, 2.001.

There are cases where computing the limit is not so obvious. For example, consider

$$y = \frac{\sin x}{x}$$

What is the limit of y as x goes to 0? If you test with calculator, the result seems to be 1.

Two applications of limits: find the slope of the tangent line to a curve, and find the average velocity on a time interval.

Slope of tangent line at P is approximately the slope of the secant PQ where Q lies on the curve and is close to P. Here, one quantity is the x coordinate of Q and the other quantity is the slope of PQ.

Average velocity: as you drive, your speed is reflected on the speedometer of your car. It seems to respond real-time to your driving. The number you see is actually the average velocity on a very short interval of time. It is almost an "instantaneous" velocity.