Lecture 9

Tuesday, September 17, 2024 10:25 AM

Work on Problem 4 of the previous worksheet.

We have learned how to find limits numerically and graphically. Both methods have their own risks.
Today, we will find limits algebraically - the most reliable method.

Reminder: a function f is said to be continuous at a if $\lim_{x\to a} f(x) = f(a)$.
Most functions are continuous at every point a in its domain. These functions include:

- The power functions $f(x) = x^n$, for $n = 1,2,3, \ldots$
- The radical functions $f(x) = \sqrt[n]{x}$ for $n = 2,3,4, \ldots$
- The trigonometric functions
- ...

Algebraic rules of limit:

- Sum rule: $\lim_{x\to a}(f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$ (if both exist)
- Product rule: $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$ (if both exist)
- Quotient rule: $\lim_{x\to a} \frac{f(x)}{g(x)} = (\lim_{x\to a} f(x))/(\lim_{(x\to a)} g(x))$ (if both exist)

Example:
Find $\lim_{x\to 1} \frac{x}{x^2+1}$

By quotient rule,
$\lim_{x\to 1} \frac{x}{x^2+1} = \frac{\lim_{x\to 1} x}{\lim\limits_{x\to 1}(x^2+1)} = \frac{1}{\lim\limits_{x\to 1}(x^2+1)}$
By sum rule,
$\lim\limits_{x\to 1}(x^2+1) = \lim\limits_{x\to 1} x^2 + \lim\limits_{x\to 1} 1 = 1^2 + 1 = 2$
Therefore,
$$\lim_{x\to 1} \frac{x}{x^2+1} = \frac{1}{2}$$