Lecture 1

Wednesday, September 4, 2024 12:13 AM

Calculus: 1, 2, 3 (Multivariable)

Analysis (introduction): 1, 2

Math 441 Math 442

analysis
- Real analysis
- Numerical analysis
- Complex analysis
- functional analysis
- nonlinear analysis

In Math 441, we revisit many familiar topics in Calc I and II, such as limits, continuity, derivative, integral. But we are more focus on building a firm theoretical foundation rather than computing skills. We emphasize more on logic and proof.

<u>Ex</u>

$$\lim_{x \to \infty} \frac{3x^2}{2x^2+1} = ?$$

We start our foundation with the number system. This is a long story, where we can start with natural numbers and some axioms to extend them to integers, rationals, reals. However, due to the limited time of the course, we will take a shortcut. We will start directly with real numbers.

Note that we not only want to define what real numbers are, but also how they interact with each other. Think about us as children of God (set of elements) - this title associates with some relations, responsibility we have with one another.

The set of real numbers is now defined as an ordered field:

$(\mathbb{R}, +, \cdot, <)$ is defined by axioms:

- If $x, y \in \mathbb{R}$ then $x+y \in \mathbb{R}$
- $x+y = y+x$ (commutativity)
- $x+(y+z) = (x+y)+z$ (associativity)
- $x+0 = x$ (neutral element)
- $x+(-x) = 0$ (inverse element)

properties of addition

- If $x, y \in \mathbb{R}$ then $xy \in \mathbb{R}$
- $xy = yx$
- $x(yz) = (xy)z$
- $x \cdot 1 = x$
- $x x^{-1} = x^{-1} x = 1$

prop. of multiplication

- $x(y+z) = xy + xz$
- $(y+z)x = yx + zx$

interaction between addition and multiplication

field property

order
- For $x, y \in \mathbb{R}$, one of the 3 scenarios must be right:

$$x<y, \quad y<x, \quad x=y$$

also write $x>y$

- If $x<y$ and $y<z$ then $x<z$

how order interacts with "+" and "·"
- If $x<y$ then $x+z<y+z$
- If $x, y>0$ then $xy>0$

- Every nonempty subset $E \subset \mathbb{R}$ has a least upper bound. (Completeness property)

The last axiom distinguishes the rational numbers from the real numbers.

Ex show that if $x<y$ and $z<t$ then $x+z<y+t$.

$x<y$, so $x+z<y+z$

Need to show $y+z<y+t$. This is true because $z<t$.

Ex Show that $0x=0$

$$0x=(0+0)x=0x+0x \longrightarrow 0x=0$$

Ex Show that $-x=(-1)x$

$$(-1)x+x=(-1)x+1\cdot x=[(-1)+1]x=0x=0$$