

Lecture 13

Friday, October 4, 2024 1:01 PM

Example of showing limit by ε - δ definition:

$$\lim_{x \rightarrow 4} \sqrt{x} = 2$$

Let $\varepsilon > 0$. We want to find $\delta = \delta_\varepsilon > 0$ such that $|\sqrt{x} - 2| < \varepsilon$ as long as $|x - 4| < \delta$, $x \neq 4$. Assume that $|x - 4| < \delta$, $x \neq 4$. Observe that

$$\sqrt{x} - 2 = \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{\sqrt{x} + 2} = \frac{x - 4}{\sqrt{x} + 2}$$

Thus,
$$|\sqrt{x} - 2| = \frac{|x - 4|}{\sqrt{x} + 2} \leq \frac{|x - 4|}{2}.$$

To ensure that $|\sqrt{x} - 2| < \varepsilon$, we need to make sure that $|x - 4| < 2\varepsilon$.

We do so by choosing $\delta = 2\varepsilon$.

Presentation time....