Lecture 16

Friday, October 11, 2024 12:05 AM

Theorem: If a function $f: [a, b] \to R$ is continuous then it is bounded.
The assumption that f is continuous is crucial. You can find a discontinuous on $[0,1]$ that is unbounded.

Proof: Suppose by contradiction that f is unbounded. Then there exists a sequence (x_n) in $[0,1]$ such that $|f(x_n)| \to \infty$. Because (x_n) is bounded, by B-W theorem, there exists a convergent subsequence (x_{n_k}). Let x_0 be the limit. Note that $a \leq x_0 \leq b$ because $a \leq x_{n_k} \leq b$ for all k. By the continuity of f, $\lim f\left(x_{n_k}\right) = f(x_0)$. Which is a contradiction because $f(x_{n_k})$ is unbounded.