Lecture 5

Friday, September 13, 2024 9:37 AM

We know that sup A is defined as the lowest upper bound of A. It exists if A is nonempty and bounded from above. Relax the notion of supremum and infimum to accommodate the case A is empty or unbounded:

Sup $\emptyset = -\infty$
Sup A $= \infty$ if A is nonempty and not bounded from above

Inf $\emptyset = \infty$
Sup A$=-\infty$ if A is nonempty and not bounded from below

Why do these conventions make sense?

<u>Important property</u>: if function f and g defined on D satisfy $f(x) \leq g(x) \forall x \in D$ then $\sup f \leq \sup g$ and $\inf f \leq \inf g$.

In principal, to show that $\sup f \leq M$, we need to show that $f(x) \leq M$ for all $x \in D$

Problem for extra credit:
Suppose $f(x) \leq g(x) \forall x \in D$. Can we conclude that $\sup f < \sup g$? Prove or give a counter example.

Presentations on HW 1.1 and 1.2