Lecture 6

Monday, September 16, 2024 9:44 AM

Sequence is a function $a : \mathbb{N} \to \mathbb{R}$

$a(1)$ is written as a_1

$a(n)$ is written as a_n

<u>Def</u> $\lim_{n\to\infty} a_n = a$ if the following is true.

$$\forall \varepsilon > 0,\ \exists N = N(\varepsilon) : \ |a_n - a| < 1 \quad \forall n > N$$

<u>Ex</u> $a_n = \dfrac{n}{n+1}$

Show that $\lim_{n\to\infty} a_n = 1$.

<u>Ex</u> $a_n = \dfrac{n^2+1}{n^2-2n}$

Show that $\lim_{n\to\infty} a_n = 1$.

$$a_n - 1 = \frac{n^2+1}{n^2-2n} - 1 = \frac{2n+1}{n^2-2n}$$

For $n \geq 4$:

$$|a_n - 1| = \frac{2n+1}{n^2-2n} < \frac{2n+2}{n^2-2n-3} = \frac{2(n+1)}{(n+1)(n-3)} = \frac{2}{n-3}$$

To guarantee that $|a_n - 1| < \varepsilon$, we will require $\frac{2}{n-3} < \varepsilon$.

Choose $N = \max\{4, \frac{2}{\varepsilon} + 3\}$.

Sequence
- bounded: $|a_n| \le M \ \forall n$
- increasing: $a_n \le a_{n+1}$
- decreasing: $a_n \geq a_{n+1}$

Subsequence:

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8 \quad a_9 \quad a_{10} \quad a_{11} \quad a_{12} \quad a_{13} \quad a_{14} \quad a_{15} \quad a_{16}$

$a_{n_1} \quad a_{n_2} \quad a_{n_3} \quad a_{n_4} \quad a_{n_5} \quad a_{n_6} \quad a_{n_7} \quad a_{n_8}$

$n_1 = 2 \qquad n_5 = 10$

$n_2 = 4 \qquad n_6 = 11$

$n_3 = 5 \qquad n_7 = 13$

$n_4 = 8 \qquad n_8 = 16$

A subsequence is a sequence drawn from another sequence.

A subsequence of a subsequence is still called a subsequence, not a subsubsequence.

Property: If (a_{n_k}) is a subsequence of (a_n) and $\lim a_n = a$, then

$\lim\limits_{k \to \infty} a_{n_k} = a$.

Ex $\quad a_n = \cos(n\pi)$

Subsequence (a_{2n}): $a_2, a_4, a_6, a_8, \ldots \qquad a_{2n} = \cos(2n\pi) = 1$ for all n

subsequence (a_{2n+1}): $a_1, a_3, a_5, a_7, \ldots \qquad a_{2n+1} = \cos((2n+1)\pi) = -1$ for all n.

$\lim a_{2n} = 1$, $\lim a_{2n+1} = -1$. Therefore, (a_n) does not converge.