Lecture 2

Friday, September 5, 2025 8:57 AM

Goals:

- Matrix representation of a linear system
- Elementary row operations as a means for transform a linear system to an equivalent but easier (simpler) linear system.

Three elementary row operations:

1) (Replacement) Replace one row by the sum of itself and a multiple of another row.
2) (Interchange) Interchange two rows.
3) (Scaling) Multiply all entries in a row by a nonzero constant.

Example 1:

$$\begin{cases} x + 2y + 3z = 9 \\ 2x - y + z = 8 \\ 3x - z = 3 \end{cases}$$

Example 2:

$$\begin{cases} x_2 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 3 \end{cases}$$

A rectangular matrix is in **echelon form** (or row echelon form) if it has the following three properties:

1) All nonzero rows are above any rows of all zeros.
2) Each leading entry of a row is in a column to the right of the leading entry of the row above it.
3) All entries in a column below a leading entry are zeros.

If a matrix in echelon form satisfies the following additional conditions, then it is in **reduced echelon form** (or reduced row echelon form):

4) The leading entry in each nonzero row is 1.
5) Each leading 1 is the only nonzero entry in its column.