Final exam: Some problems for review

The exam will be held in the regular classroom (Badgley 142) from 3 PM to 5 PM on Monday, June 12, 2023. The material covered is Section 10.4 - 10.7 and 11.2, 11.3, 11.4, 11.7, 11.8, 11.9 (dot product only). It is a closed book exam. You can bring the trigonometric cheat sheet I handed out earlier. In addition, you can bring a 4" x 6" single-sided handwritten note card. A scientific calculator is allowed. Graphing/ programmable/ transmittable calculators are not allowed.

You should review the homework problems, the examples given in the textbook and in the lectures. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Prove some simple trigonometric identities using the cheat sheet.
- Graph trigonometric functions using familiar transformations (horizontal/ vertical translation, scaling, reflection).
- Find the period, amplitude, phase shift and vertical shift of a given function.
- Find arcsin, arccos, arctan.
- Solve trigonometric equations by converting them into basic forms.
- Use the Law of Sines and/or Cosines to find the missing sides and angles of a triangle.
- Convert between Cartesian coordinates and polar coordinates.
- Perform basic arithmetics on complex numbers using polar or standard forms.
- Perform basic arithmetics on vectors.

Additional problems to practice:

1) Prove the identity
$$8\cos^4(\theta) = \cos(4\theta) + 4\cos(2\theta) + 3$$

2) Write $\cos\theta - \sin\theta$ as a product.

3) Graph the function $f(x) = -\sin\left(x + \frac{\pi}{3}\right)$.

4) Find the period, amplitude, phase shift and vertical shift of the function
$$f(x) = -\frac{3}{2}\cos\left(2x + \frac{\pi}{3}\right) + 4$$

5) Find the exact value of

   (a) $\sin(\arcsin 2)$

   (b) $\sin(\arcsin\left(\frac{1}{2}\right))$

   (c) $\arcsin(\sin\left(\frac{\pi}{6}\right))$

   (d) $\arcsin(\sin\left(-\frac{\pi}{3}\right))$

   (e) $\arcsin(\sin\left(\frac{11\pi}{4}\right))$

6) Find the exact value of

(a) $\cos(\arccos(-\frac{1}{2}))$

(b) $\arccos(\cos\left(\frac{2\pi}{3}\right))$

(c) $\arccos(\cos\left(\frac{13\pi}{2}\right))$

(d) $\arccos(\cos\left(-\frac{\pi}{6}\right))$

7) Find the exact value of

(a) $\tan(\arctan(\frac{1}{2}))$

(b) $\arctan(\tan\left(-\frac{\pi}{4}\right))$

(c) $\arctan(\tan(\pi))$

(d) $\arctan(\tan\left(\frac{\pi}{2}\right))$

8) Solve the following equations

(a) $\cos(2x) = -\sin x$

(b) $\sin(2x) = \tan x$

9) Use the Law of Sines and/or Cosines to find the missing sides and angles of a triangle.

(a) $\alpha = 6°$, $a = 57$, $c = 100$

(b) $\beta = 102°$, $b = 16.75$, $c = 18$

(c) $a = 153$, $\beta = 8.2°$, $c = 153$

(d) $a = 7$, $b = 10$, $c = 13$

10) Find $(\sqrt{3} + i)^5$ in both standard form and polar form.

11) Consider the points $A(0, 3)$, $B(-3, -1)$, $C(2, -2)$. Determine the measure (in decimal degree) of the acute angle between the y-axis and the line BC.