Lecture 1

* Introduction

This course is a preparatory course for Calculus. The main topics to be covered are:

- Exponential functions
- Logarithm functions
- Trigonometric functions
- Inverse trigonometric functions
- Complex numbers

Exponential functions :

$$2^2 = 2 \times 2$$

$$2^4 = 2 \times 2 \times 2 \times 2$$

$$2^{-4} = \frac{1}{2^4}$$

$$2^{\frac{1}{2}} = \sqrt{2} \ . \quad \text{Why?} \qquad 2^{\frac{1}{2}} 2^{\frac{1}{2}} = 2^{\frac{1}{2} + \frac{1}{2}} = 2^1 = 2$$

$$\sqrt{2} \, \sqrt{2} = 2$$

How about $2^{\frac{1}{3}}$?

$$2^{\frac{1}{3}} \, 2^{\frac{1}{3}} \, 2^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 2^{1} = 2$$

Thus, $2^{\frac{1}{3}} = \sqrt[3]{2}$.

How about $2^{\frac{2}{3}}$?

$$2^{\frac{2}{3}} \, 2^{\frac{2}{3}} \, 2^{\frac{2}{3}} = 2^{\frac{2}{3}+\frac{2}{3}+\frac{2}{3}} = 2^{2}$$

Thus, $2^{\frac{2}{3}} = \sqrt[3]{2^{2}}$

In general, $a^{\frac{m}{n}} = \sqrt[n]{a^{m}}$.

How about $2^{\sqrt{2}}$?

$$\sqrt{2} \approx 1.4142\ldots$$

$$2^{\sqrt{2}} \approx 2^{1.4} = 2^{\frac{7}{5}} = \sqrt[5]{2^{7}}$$

$$2^{\sqrt{2}} \approx 2^{1.41} = 2^{\frac{141}{100}} = \sqrt[100]{2^{141}}$$

$$2^{\sqrt{2}} \approx 2^{1.414} = 2^{\frac{1414}{1000}} = \ldots$$

In general, we can define 2^{x} for any real number x.

2^{x} is an exponential function. 2 is the base and x is the exponent.