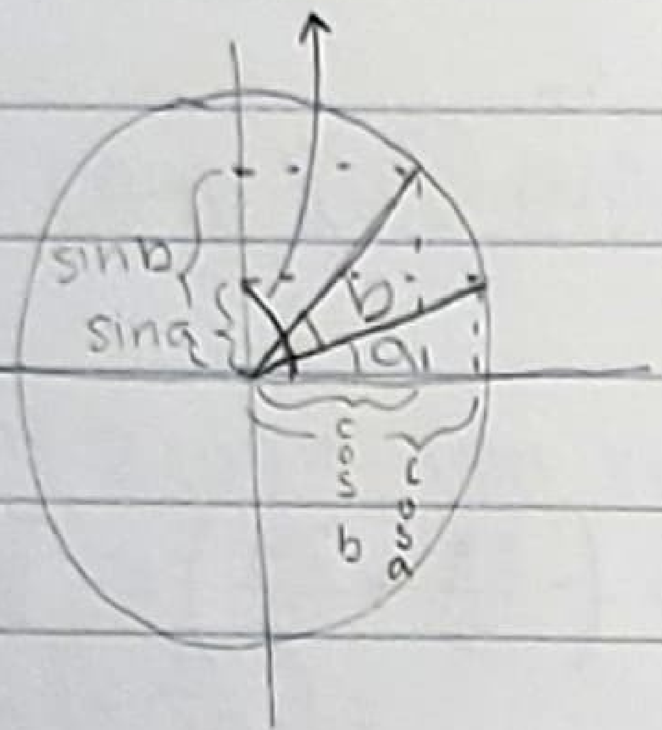
Sum Identities

08 may 2023

- $\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$



$b = \pi$: we know $\sin(\pi + a) = -\sin(a)$

plug in $\longrightarrow \sin(a)\underbrace{\cos(\pi)}_{-1} + \underbrace{\cancel{\sin(\pi)}}_{0}\cos(a) = -\sin(a)$

$b = \pi/2$: $\sin(a)\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1}\cos(a) = \cos(a)$

- Double angle identity

to find sine:

$\sin(a+a) = \sin a \cos a + \sin a \cos a$

$\rightarrow \underline{\sin(2\theta) = 2\sin(\theta)\cos(\theta)}$

to find cosine:

$\cos(a+b) = ??$ $\rightarrow$ plug into sum identity but with $b + \pi/2$

$\sin(a + b + \pi/2) = \sin(a)\underbrace{\cos\left(b + \frac{\pi}{2}\right)}_{\text{-b and } b+\pi/2 \text{ are compliment angles}} + \underbrace{\sin(b + \pi/2)}_{\cos b}\cos(a)$

$\cos\left(b + \frac{\pi}{2}\right) = \sin(-b) \rightarrow$ co-func identity

$\quad\quad = -\sin(b) \rightarrow$ NAI

$\rightarrow \underline{\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)}$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right)$

$\boxed{\text{PRACTICE}}$

▷ a) $\cos(75^\circ) = \cos(45^\circ + 30^\circ) = \cos(45)\cos(30) - \sin(45)\sin(30)$

$\left(\frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \quad \frac{1}{2}\right)$

$0.6124 \qquad 0.3536$

$\boxed{\cos(75^\circ) = 0.2588}$

▷ b) $\sin(105^\circ) = \sin(60^\circ + 45^\circ) = \sin(60)\cos(45) + \cos(60)\sin(45)$

$\left(\frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2} \quad \frac{\sqrt{2}}{2}\right)$

$\boxed{\sin(105^\circ) = 0.9659}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \qquad \frac{\sqrt{6}+\sqrt{2}}{4}$

▷ c) $\cos\left(\frac{13\pi}{12}\right) = -\cos\left(\frac{13\pi}{12} - \pi\right) = -\cos(\pi/12) \to \left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$-\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) =$

$\boxed{\text{Proof of trig identities}}$

09 may 2023

① show that : $(\cos\theta + \sin\theta)^2 = 1 + \sin(2\theta)$

• remember that $\cos^2\theta + \sin^2\theta = 1$ (pytagorean theorum $a^2+b^2=c^2$)

↳ also $(\cos\theta)^2 = \cos^2\theta$ or $\tan^3 x = (\tan x)^3$

so... $(\cos\theta + \sin\theta)^2 \longrightarrow (\cos\theta + \sin\theta)(\cos\theta + \sin\theta) \longrightarrow \underline{\cos^2\theta} + \cos\theta\sin\theta + \sin\theta\cos\theta + \underline{\sin^2\theta}$

$\longrightarrow \cos^2\theta + \sin^2\theta = 1$, $\underbrace{\cos\theta\sin\theta + \sin\theta\cos\theta}_{\text{angle sum + dif identities}} = \sin(2\theta)$ DAI

answer : ∴ we know that

$(\cos\theta + \sin\theta)^2 = (\cos\theta + \sin\theta)(\cos\theta + \sin\theta) \to$ foil

$= \cos^2\theta + \sin\theta\cos\theta + \cos\theta\sin\theta + \sin^2\theta$

$= (\cos^2\theta + \sin^2\theta) + 2\sin\theta\cos\theta$

[remember: $\cos^2\theta = (\cos\theta)^2$

$\sin^2\theta = (\sin\theta)^2$]

by the pythagorean theorum,

$\cos^2\theta + \sin^2\theta = 1$

by the double angle identity,

$2\sin\theta\cos\theta = \sin(2\theta)$

therefore,

$(\cos\theta + \sin\theta)^2 = 1 + \sin(2\theta)$

② $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$ Hint: $3\theta = 2\theta + \theta$

we know that,

$\sin(3\theta) = \sin(2\theta + \theta)$

by the angle sum identity,

★ $\sin(2\theta + \theta) = \sin(2\theta)\cos\theta + \cos(2\theta)\sin\theta$

by the double angle identity,

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

→ $★ = (2\sin\theta\cos\theta)\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \cos^2\theta\sin\theta - \sin^3\theta$ (★★

by the pythagorean theorum,

$\cos^2 = 1 - \sin^2\theta$

$★★ = 2\sin\theta(1-\sin^2\theta) + (1-\sin^2\theta)\sin\theta - \sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - \sin^3\theta - \sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

therefore,

$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$\boxed{\text{Proof when both sides are complicated}}$ 11 may 2023

$$(1+\tan^2 x)^3 = \frac{2}{(1+\cos 2x)\cos^4 x}$$

we know that,

$$\left(1+\frac{\sin^2 x}{\cos^2 x}\right)^3 = \frac{2}{(2\cos^2 x)\cos^4 x}$$

here we have used the identities:

① $\tan = \sin/\cos$

② $\cos(2x) = 2\cos^2 x - 1$ (DAI)

$$\left(\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\right)^3 = \frac{2}{2\cos^6 x}$$

$$\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)^3 = \frac{1}{\cos^6 x}$$

Now use pythagorean thereum

$\cos^2 x + \sin^2 x = 1$

→ $(1/\cos^2 x)^3 = 1/\cos^6 x$

$$\frac{1}{\cos^6 x} = \frac{1}{\cos^6 x}$$

DONE

therefore,

$$(1+\tan^2 x)^3 = \frac{2}{(1+\cos 2x)\cos^4 x}$$