Lecture 2

Tuesday, April 4, 2023 12:23 AM

* Questions ...

a^x is called the base a exponential equation.

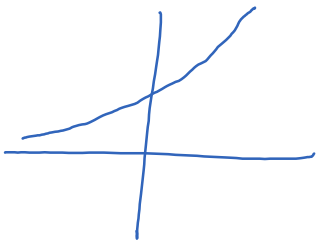
We only consider $a>0$. Here x can be any number.

If $a>1$, the function $f(x)=a^x$ is increasing.

$f(x)\to\infty$ as $x\to\infty$

$f(x)\to 0$ as $x\to-\infty$

$f(0)=1$

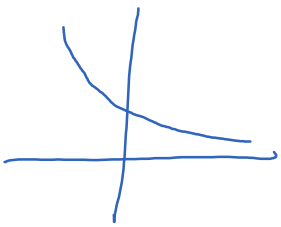


If $0<a<1$, the function $f(x)=a^x$ is decreasing.

$f(x)\to 0$ as $x\to+\infty$

$f(x)\to\infty$ as $x\to-\infty$

$f(0)=1$.



Use Geogebra.org to visualize the functions 2^x, 3^x, $\left(\frac{1}{2}\right)^x$, 1^x.

Algebraic properties:

$$a^x a^y = a^{x+y}$$

$$a^{-x} = \frac{1}{a^x}$$

$$\frac{a^x}{a^y} = a^{x-y} \qquad (ab)^x = a^x b^x$$

$$(a^x)^y = a^{xy} \qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

Ex Simplify $\left(\frac{9}{16}\right)^{-1/2}$.

$$\left(\frac{9}{16}\right)^{-1/2} = \frac{9^{-1/2}}{16^{-1/2}} = \frac{\frac{1}{9^{1/2}}}{\frac{1}{16^{1/2}}} = \frac{\frac{1}{\sqrt{9}}}{\frac{1}{\sqrt{16}}} = \frac{\frac{1}{3}}{\frac{1}{4}} = \frac{4}{3}.$$

Note: $2^x \neq x^2$

- 2^x: exponential function
- x^2: power function

Graph show that $2^x > x^2$ when x is large.

The equation $2^x = x^2$ has exactly 3 solutions.

Two special bases

* $a = 10$: 10^x → common base

* $a = e \approx 2.71\ldots$: e^x → natural base

The logarithm is the inverse of the exponential function.

$$a^x \longleftrightarrow \log_a x$$

That is,

$$a^{\log_a x} = x$$

$$\log_a(a^x) = x$$

Note:

$$\log_a x \neq \frac{1}{a^x}$$