(ex) D: $[-1,1]$ R: $[-\pi/2, \pi/2]$

a) $\arcsin(1) = \boxed{\frac{\pi}{2}}$

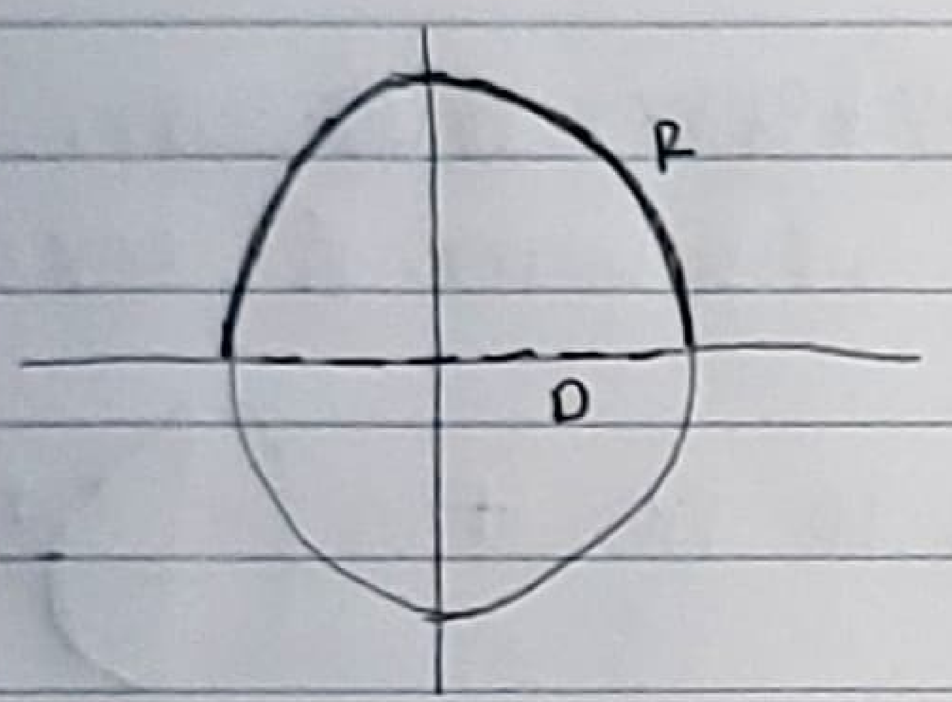
b) $\arcsin(0) = \boxed{0}$, π is outside range

c) $\arcsin(1/2) = \boxed{\frac{\pi}{6}}$, $\frac{5\pi}{6}$ is out of range

d) $\arcsin(\sqrt{2}/2) = \boxed{\frac{\pi}{4}}$

e) $\arcsin(-\sqrt{3}/2) =$ $\sin(-\pi/3) = -\sqrt{3}/2 = \boxed{-\pi/3}$

Other inverses



arccos

D: $[-1, 1]$
R: $[0, \pi]$

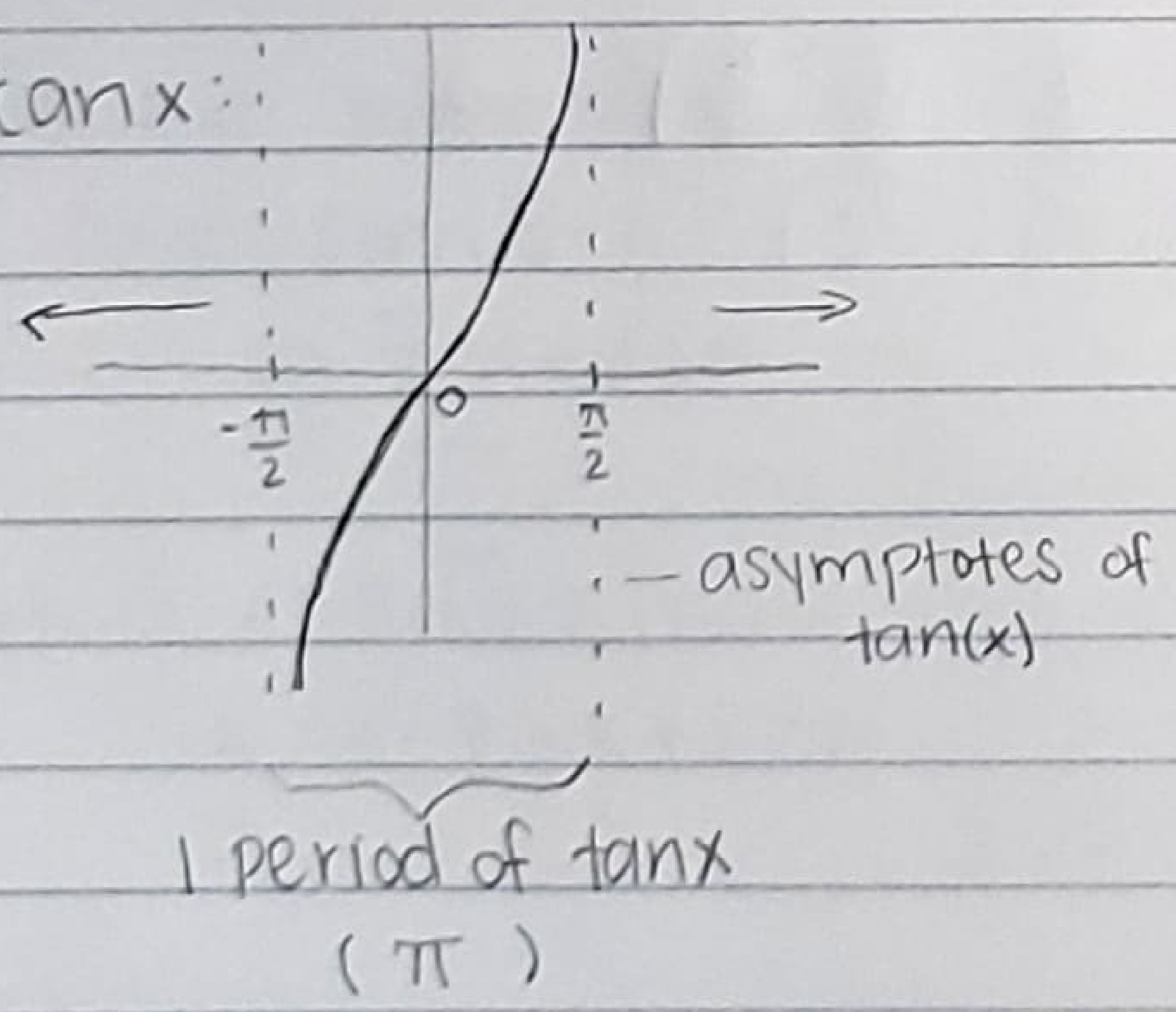
$\arccos(-1/2) = \frac{2\pi}{3}$
$\arccos(1) = 0$
$\arccos(0) = \pi/2$
$\arccos(-1) = \pi$

Solving equations

18 may 2023

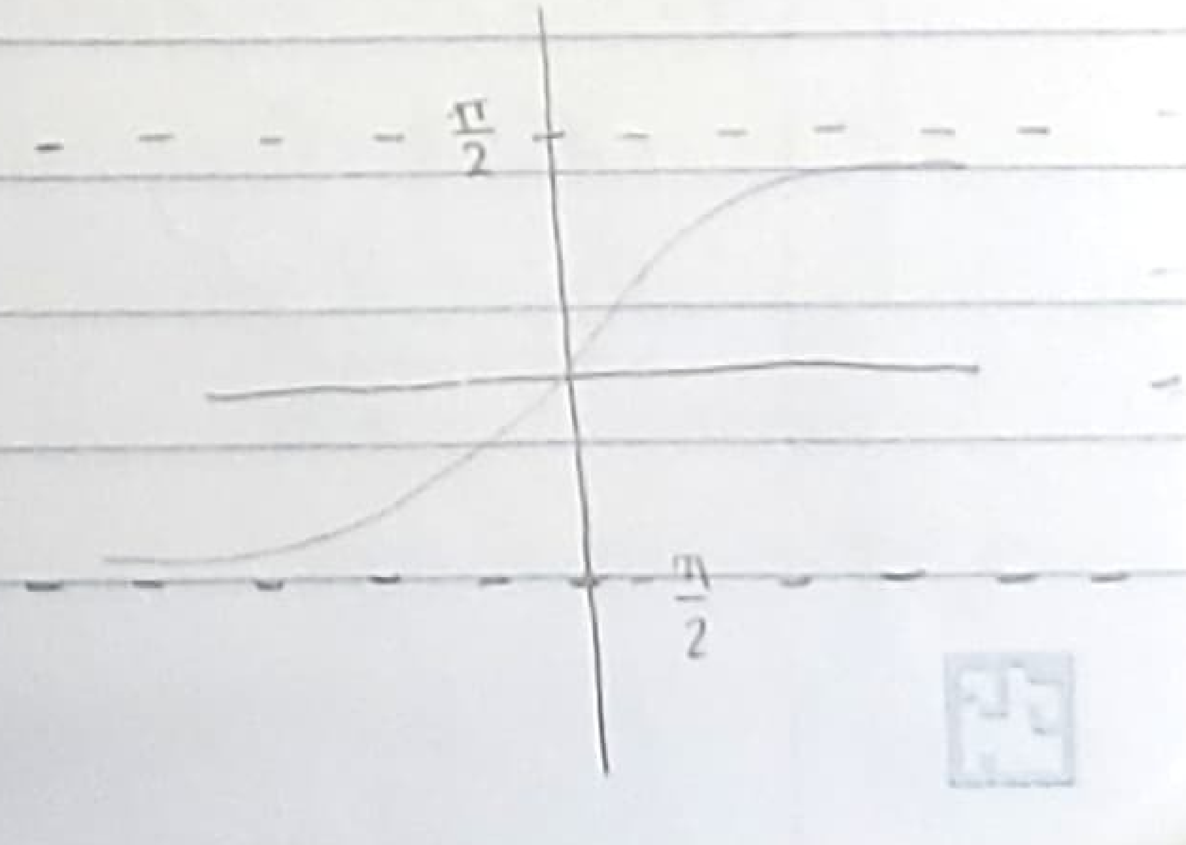
- Recall:
  - $\sin x$, $\cos x$, $\sec x$, $\csc x$, are periodic with period 2π
  - $\tan x$, $\cot x$ are periodic with period π

tanx:



As $x \to \frac{\pi}{2}^-$, $\tan(x) \to \infty$
$x \to -\frac{\pi}{2}^+$, $\tan(x) \to -\infty$
and $\tan(0) = 0$
and $\tan(x)$ is increasing from $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

★ $\arctan(x)$ is the inverse of $\tan(x)$ ★
- D: $(-\infty, \infty)$
- R: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



PRACTICE

① if $\sin\theta = \frac{x}{2}$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,
what is $\theta + \sin(2\theta)$ in terms of x?

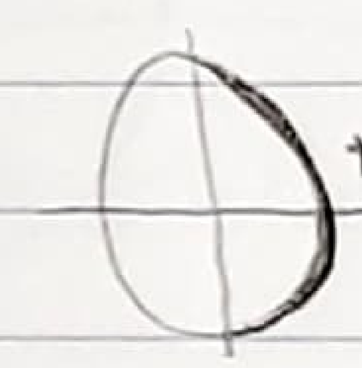
$\theta + (2\sin\theta\cos\theta)$

to get $\sin\theta = x/2$ into terms of θ, use arcsin
↳ $\theta = \arcsin(\frac{x}{2})$

to turn $\sin(2\theta)$ into terms of x, use DA identity first
↳ $\sin(2\theta) = 2\sin\theta\cos\theta$, then replace $\sin\theta$ with $\frac{x}{2}$ → $(\sin\theta = \frac{x}{2})$
↳ $2(\frac{x}{2})\cos\theta$ → cancel 2's → $x\cos\theta$

to get cosθ to apply to sinθ, use pythagorean theorum
↳ $\cos^2\theta + \sin^2\theta = 1$ → $\cos^2\theta = 1 - \sin^2\theta$, replace sinθ with $\frac{x}{2}$ again
↳ $\cos^2\theta = 1 - (\frac{x}{2})^2$ → $\cos\theta = \pm\sqrt{1-(\frac{x}{2})^2}$

because θ is between $(-\frac{\pi}{2}, \frac{\pi}{2})$, cos θ has to be +



∴ $\cos\theta = \sqrt{1-(\frac{x}{2})^2}$

SO, to conclude:

θ sin cos

$$\theta + \sin(2\theta) = \arcsin(\tfrac{x}{2}) + x\sqrt{1-(\tfrac{x}{2})^2}$$

② if $\cos\theta = \frac{x}{2}$ for $\pi < \theta < \frac{3\pi}{2}$, what is $\theta + \sin(2\theta)$ in terms of x?

$\cos\theta = \frac{x}{2}$ → terms of x: $\theta = \arccos(\frac{x}{2})$ → out of range $[0, \pi]$
↳ so $0 < \theta - \pi < \frac{\pi}{2}$ is in range

$\cos(\theta - \pi) = -\cos\theta = -\frac{x}{2}$

$\theta - \pi = \arccos(-\frac{x}{2})$ ⇝ $\theta = \pi + \arccos(-\frac{\pi}{2})$

$\sin(2\theta) = 2\sin\theta\cos\theta$
$-\sqrt{1-(\frac{x}{2})^2}$ $\frac{x}{2}$ →