

Trig equations

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- Recall:

$$a^b = c \iff b = \log_a c$$

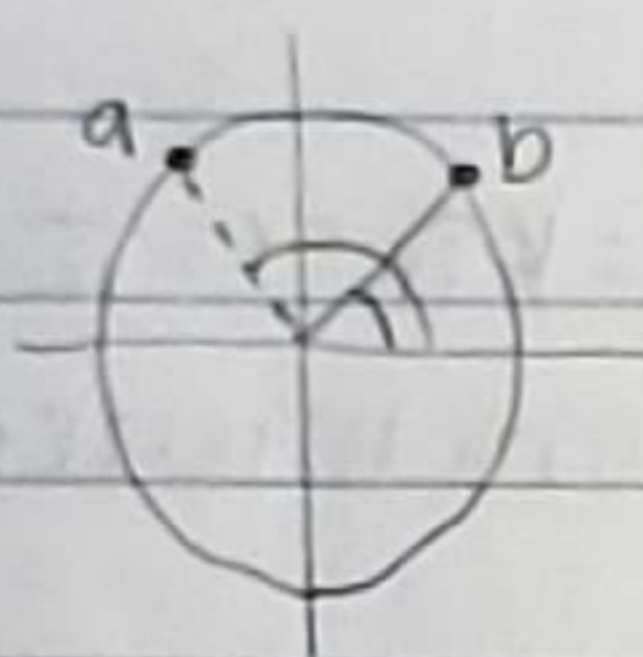
$$\log_a X = \log_a Y \iff X = Y$$

$$a^x = a^y \iff x = y$$

- Basic forms:

▷ $\sin(a) = \sin(b) \iff a = b + k2\pi$ or $a = \pi - b + k2\pi$

↳ any integer



$$a = b \quad a = 3\pi - b$$

$$a = \pi - b$$

$$a = b + 2\pi$$

ex) $\sin(x) = \frac{1}{2}$

$$\sin(x) = \sin\left(\frac{\pi}{6}\right) \rightarrow x = \frac{\pi}{6} + k2\pi \text{ or } x = \frac{5\pi}{6} + k2\pi$$

$$\sin x = 1$$

$$\sin x = \sin \frac{\pi}{2} \rightarrow x = \frac{\pi}{2} + k2\pi \text{ or } x = \frac{\pi}{2} + k2\pi \rightarrow x = \pi - \frac{\pi}{2} + k2\pi$$

cases
can be
merged
together

$$\sin x = 0$$

$$\sin x = \sin \pi \rightarrow x = \pi + k2\pi \text{ or } x = 0 + k2\pi \rightarrow x = k\pi$$

ex) solve $\sin(x) = \cos(x)$ ~~~~~ use co-func identity to turn into sin

$$\sin x = \sin\left(\frac{\pi}{2} - x\right)$$

co-term

case 1: $x = \frac{\pi}{2} - x + k2\pi$

$$-x \quad +x \quad 2x = \frac{\pi}{2} + k2\pi \Rightarrow x = \frac{\pi}{4} + k\pi$$

sup

case 2: $x = \pi - \left(\frac{\pi}{2} - x\right) + k2\pi$

$$x = \pi - \frac{\pi}{2} + x + k2\pi$$

$$-x \quad -x$$

$$0 = \pi - \frac{\pi}{2} + k2\pi \rightarrow 0 = \frac{\pi}{2} + k2\pi = \pi\left(\frac{1}{2} + 2k\right)$$

$$0 = \frac{1}{2} + 2k$$

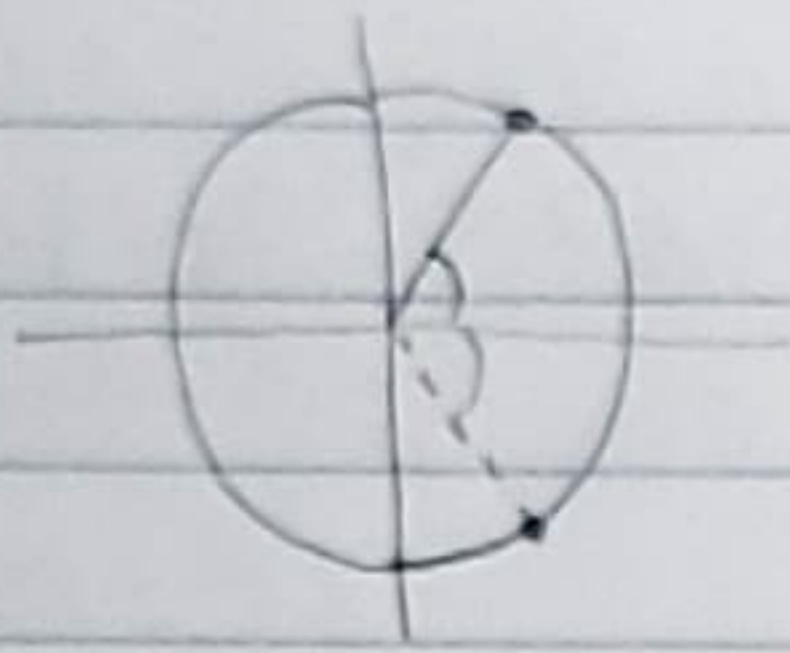
$$\rightarrow k = -\frac{1}{4}$$

not an integer

so not the answer

basic forms cont...

▷ • $\cos(a) = \cos(b) \iff a = b + k2\pi$ or $a = -b + k2\pi$



▷ • $\tan(a) = \tan(b) \iff a = b + k\pi$

⊗ $(\tan x)(\sin x - 1) = 0$

either $\tan x = 0$ or $\sin x = 1$

$\tan x = \tan 0$

$x = 0 + k\pi$

$\sin x = \sin \frac{\pi}{2}$

$x = \frac{\pi}{2} + k2\pi$

so, $x = k\pi$ or $\frac{\pi}{2} + k2\pi$

for any integer k

⊗ with interval: $\cos x = \frac{\sqrt{3}}{2}$, $x \in [-2\pi, 5\pi]$

$\cos x = \cos\left(\frac{\pi}{6}\right)$

$x = \frac{\pi}{6} + k2\pi$ or

$x = -\frac{\pi}{6} + k2\pi$

$\left\{ -2, -1, 0, 1, 2, \dots \right.$

} pick k to find angles that fit in interval