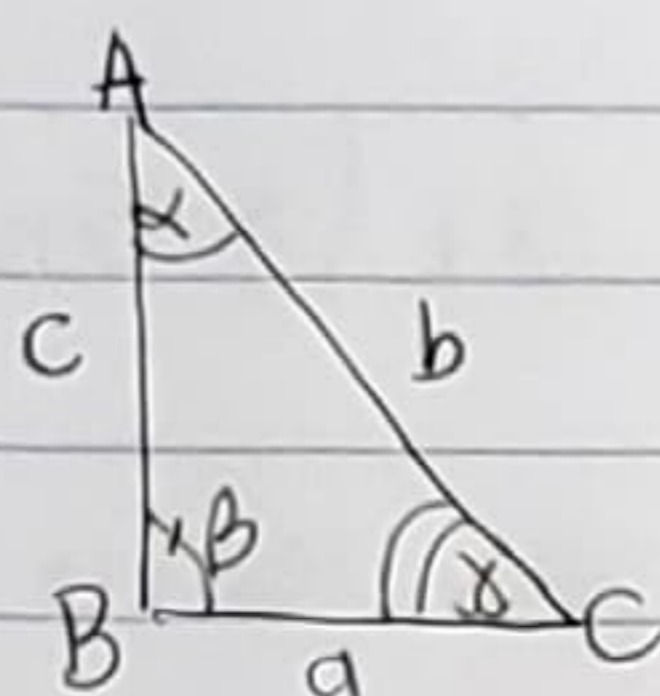


23 may 2023

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

→ difficult to show by triangles, easier to show using unit circle

$(a, \alpha)$   
 $(b, \beta)$   
 $(c, \gamma)$  } angle-side  
                              } opposite pairs



★ constraints ★

$$\alpha + \beta + \gamma = 180^\circ$$

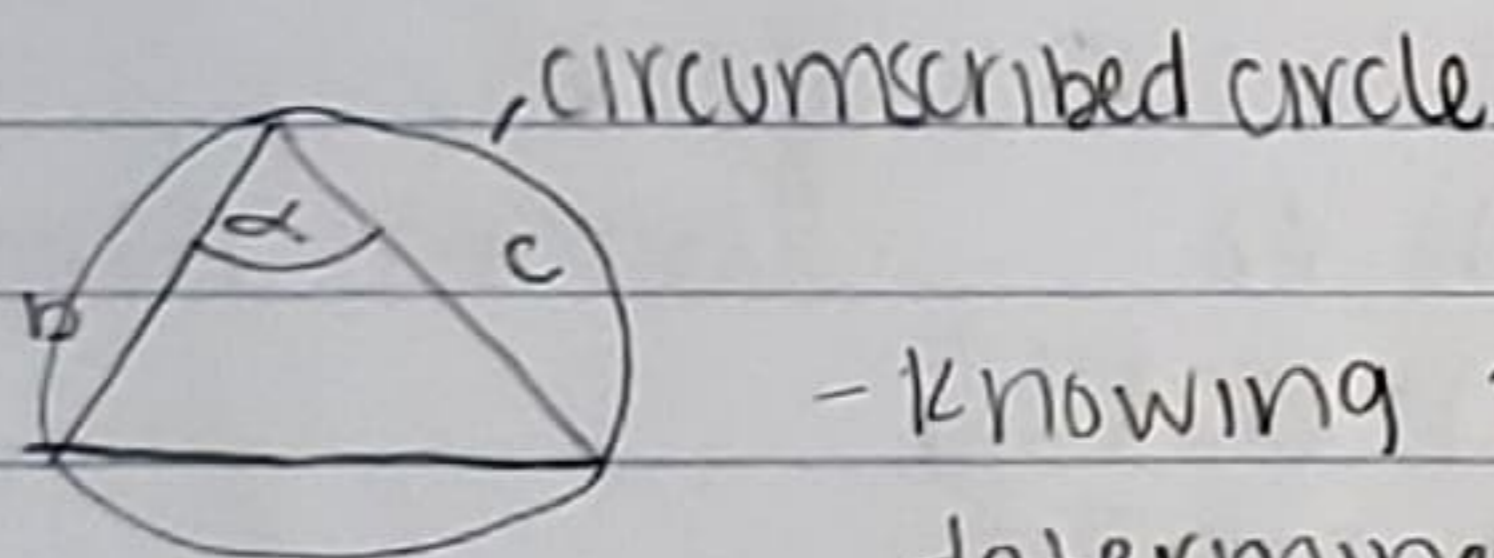
$$a + b > c$$

$$b + c > a$$

$$c + a > b$$

- observations:

- two angles determine the other angle
- two side lengths don't determine the other side length



- knowing 2 sides + the angle between them will determine the other side + other angles

famous trig iden. in a triangle

1) Law of sines:  $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = \text{diameter of the circumscribed circle}$

2) Law of cosines:

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

- observations:

- If we know  $a, \alpha, \beta$  then we know  $a/\sin \alpha$ , so then we know  $b/\sin \beta$ , so we know  $b$

$$\gamma = 180^\circ - \alpha - \beta$$

we know  $c/\sin \gamma$ , so we know  $c$

- If we know  $b, c, \alpha$ , then  $2bc \cos(\alpha) = b^2 + c^2 - a^2$

$$a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

$$a = \sqrt{b^2 + c^2 - 2bc \cos(\alpha)} \rightarrow \text{then we know } a/\sin \alpha, \text{ then we know } \sin \beta + \sin \gamma$$

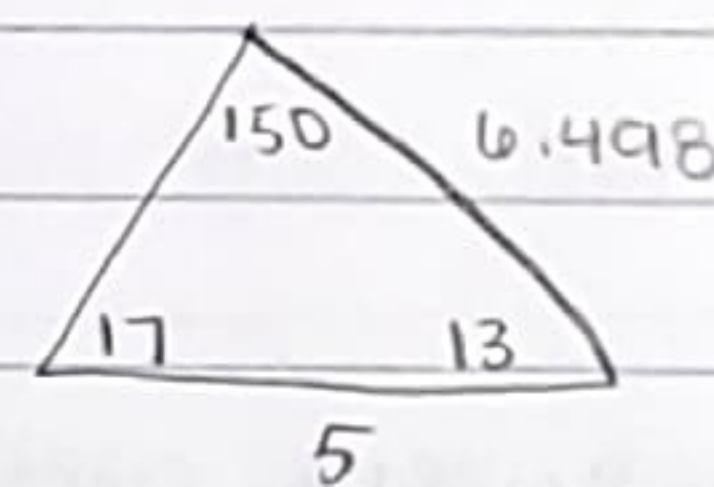
(ex) find the missing side lengths + angles

$$1) \alpha = 13^\circ \quad \beta = 17^\circ \quad a = 5$$

$$\gamma = 180 - 13 - 17 = 150$$

$$\frac{5}{\sin 13} = \frac{b}{\sin 17} = 22.22$$

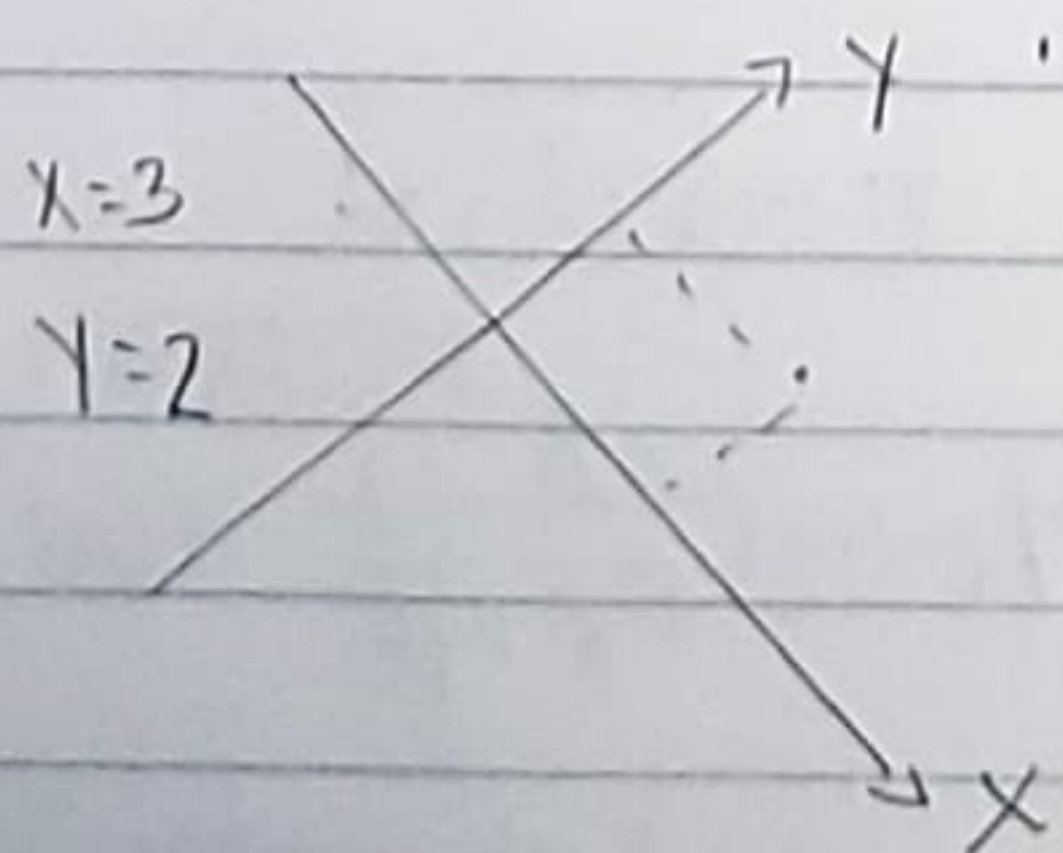
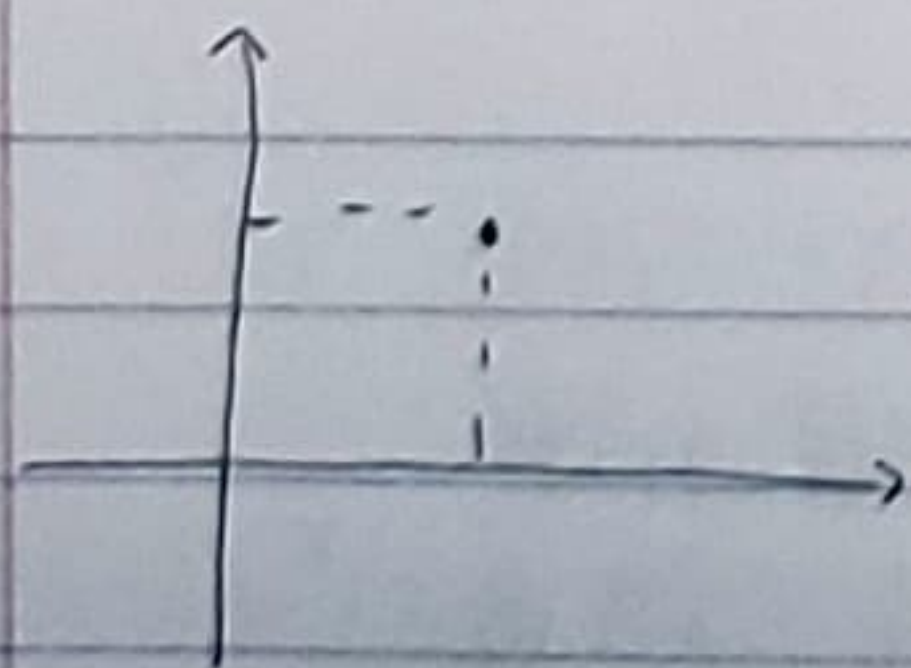
$$\begin{aligned} b &= 22.22 (\sin 17) \\ &= 22.22 (\sin 17) \\ &= 6.498 \end{aligned}$$



26 may 2023

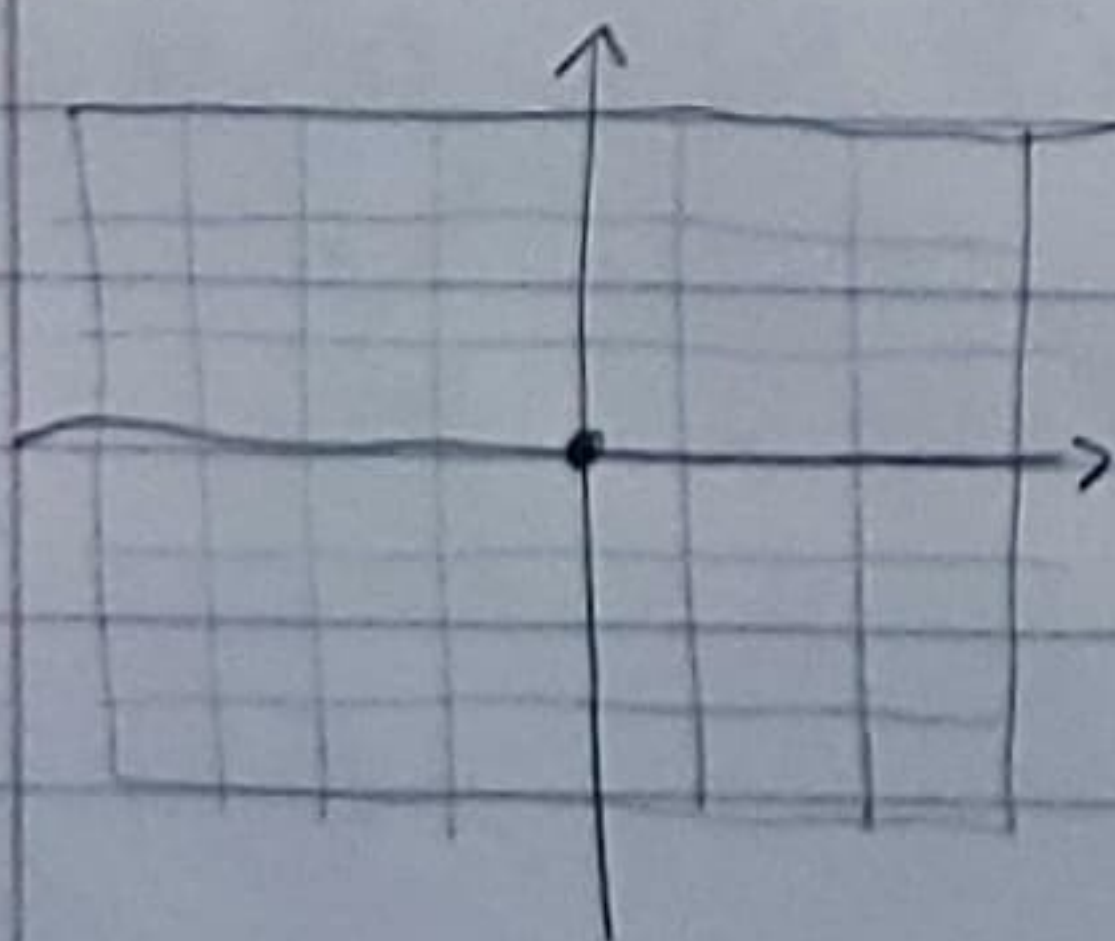
### polar coordinates

- to locate a position on a map, we give it an address



address/position depends on the coordinate system we use

(x, y)-coordinates are called cartesian coordinate



cartesian coordinate is suitable to assign an address in a rectangular shape