ex Find the polar coordinate of (r, θ) with $r > 0$, $0 \le \theta \le 2\pi$ of the point $(x, y) = (-1, \sqrt{3})$

$r = \sqrt{(-1)^2 + \sqrt{3}^2} \longrightarrow \sqrt{1 + \sqrt{3}} \longrightarrow \sqrt{4} = 2$

$\cos\theta = \frac{x}{y} = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$ $\qquad \theta = \frac{2\pi}{3} + k2\pi$ $\qquad \theta = -\frac{2\pi}{3} + k2\pi$

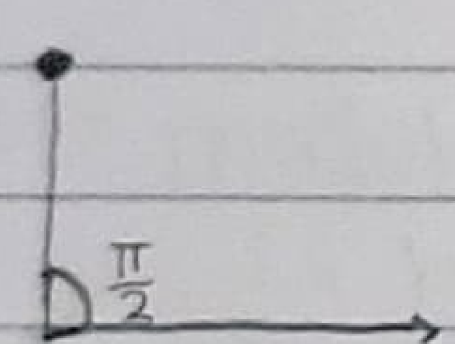
$\cos\theta = \cos\left(\frac{2\pi}{3}\right)$ $\qquad \theta = \frac{2\pi}{3}$ $\qquad \theta = \frac{4\pi}{3}$ $\qquad \boxed{\left(2, \frac{2\pi}{3}\right)}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$, so we pick $\frac{2\pi}{3}$ since $\sin\theta$ is positive

Converting examples cont

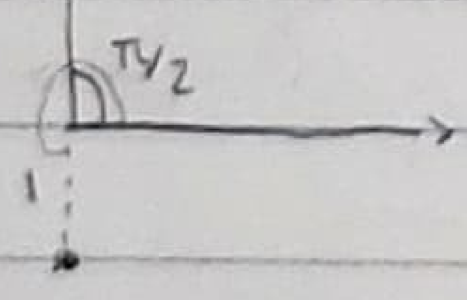
30 May 2023

$r = 1$, $\theta = \frac{\pi}{2}$



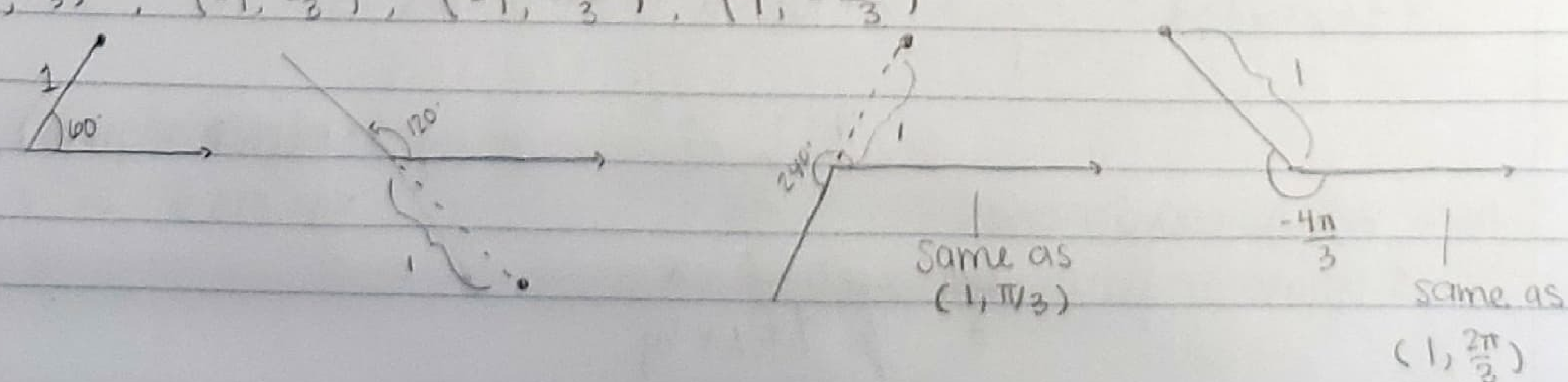
$(1, \pi/2) \longrightarrow \underbrace{+2\pi = \left(1, \frac{5\pi}{2}\right) \longrightarrow -2\pi = \left(1, \frac{3\pi}{2}\right)}_{}$ equivalent to $(1, \pi/2)$

- for graphing:
  - we allow r to be negative

$(-1, \pi/2) \longrightarrow$  but $\left(1, \frac{3\pi}{2}\right)$ is equiv. to $\left(-1, \frac{\pi}{2}\right)$

ex Draw the points with polar coordinates:

$(1, \pi/3)$, $\left(-1, \frac{2\pi}{3}\right)$, $\left(-1, \frac{4\pi}{3}\right)$, $\left(1, -\frac{4\pi}{3}\right)$



Key Points

$(-1, \frac{4\pi}{3}) \sim (1, \frac{\pi}{3})$

If you switch the sign of r, add/subtract π to get the equiv. coordinate

$(1, -\frac{4\pi}{3}) \sim (1, \frac{2\pi}{3})$

If you don't switch the sign of r but want to switch it for θ, add/subtralt 2π to get the equiv. point.

Complex numbers

01 June 2023

$Z = a+bi \longrightarrow$ standard form of complex #s

$(z = 1+3i)$ ↳ any polynomial w/ complex coefficients has complex roots

↳ a and b are real #'s $\quad \mathbb{C}$ = algebraically closed

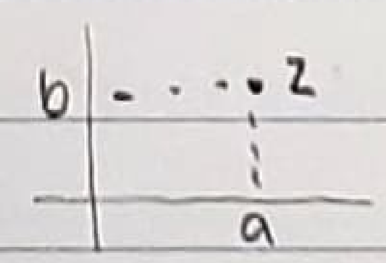
a = real part of z

b = imaginary part of z

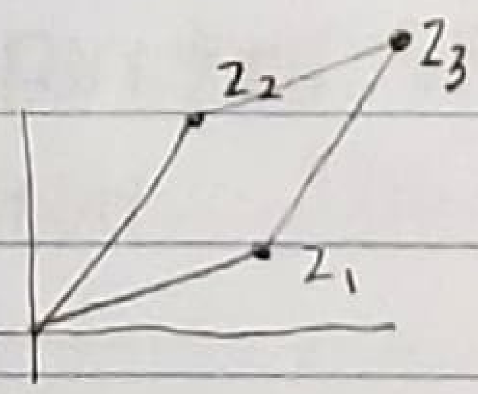
- polar form:

$$Z = re^{i\theta}$$ where (r, θ) = polar coordinate

- representations of complex numbers
  - $Z = a+bi \longrightarrow$ standard



ex $\underset{z_1}{(1+2i)} + \underset{z_2}{(3+4i)} = \underset{z_3}{4+6i}$



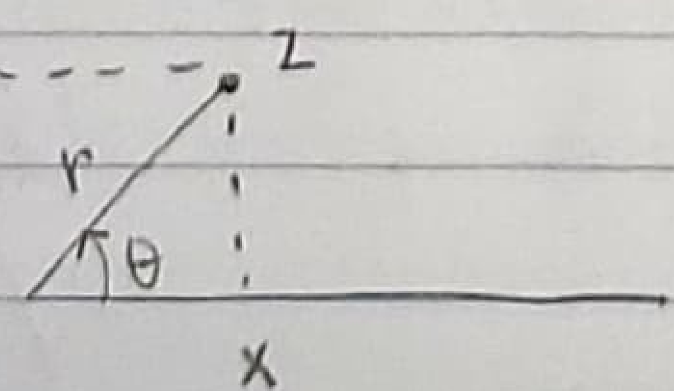
$\longrightarrow$ parallelogram rule

ex $(1+2i)(3+4i)$

$3+4i+6i+8i^2 \longrightarrow 3+10i-8 \longrightarrow -5+10i$

$i^2 = -1$

Use polar coordinates to plot multiplying



$z = r(\cos\theta + i\sin\theta)$

$= r \text{ cis } \theta$ or $r\angle\theta$

$z = x + iy$

$z = r\cos\theta + ir\sin\theta$