☆ Key points ☆

$(-1, \frac{4\pi}{3}) \sim (1, \frac{\pi}{3})$

If you switch the sign of r, add/subtract π to get the equiv. coordinate

$(1, -\frac{4\pi}{3}) \sim (1, \frac{2\pi}{3})$

If you don't switch the sign of r but want to switch it for θ, add/subtract 2π to get the equiv. point.

Complex numbers

01 June 2023

$z = a + bi$ ⟶ standard form of complex #s

$(z = 1 + 3i)$

↳ any polynomial w/ complex coefficients has complex roots

$\mathbb{C}$ = algebraically closed

↳ a and b are real #'s

a = real part of z

b = imaginary part of z

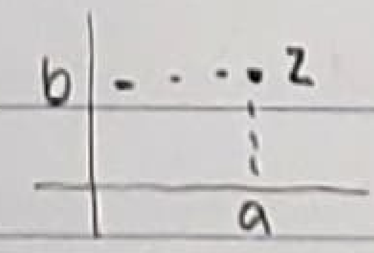
- polar form:

$$z = re^{i\theta}$$

where (r, θ) = polar coordinate

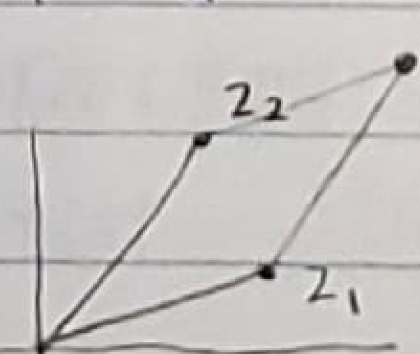
- representations of complex numbers

• $z = a + bi$ ⟶ standard



ex $(1+2i) + (3+4i) = 4 + 6i$

z_1 z_2 z_3



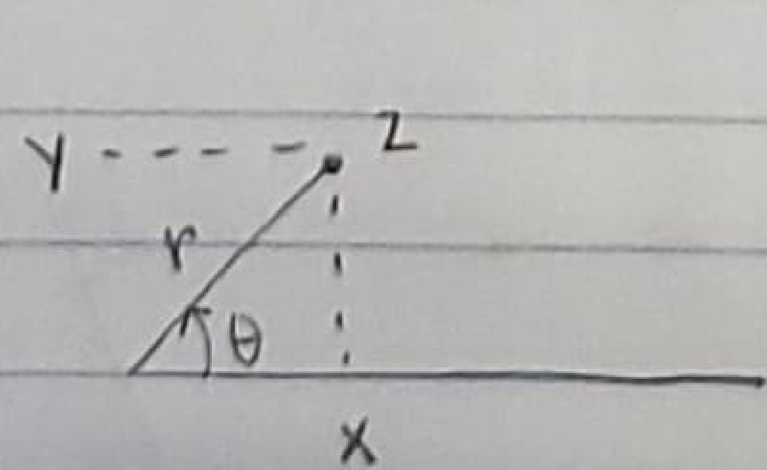
⟶ parallelogram rule

ex $(1+2i)(3+4i)$

$3 + 4i + 6i + 8i^2$ ⟶ $3 + 10i - 8$ ⟶ $-5 + 10i$

$i^2 = -1$

Use polar coordinates to plot multiplying



$z = r(\cos\theta + i\sin\theta)$

$= r \text{ cis } \theta$ or $r\angle\theta$

$z = x + iy$

$z = r\cos\theta + ir\sin\theta$

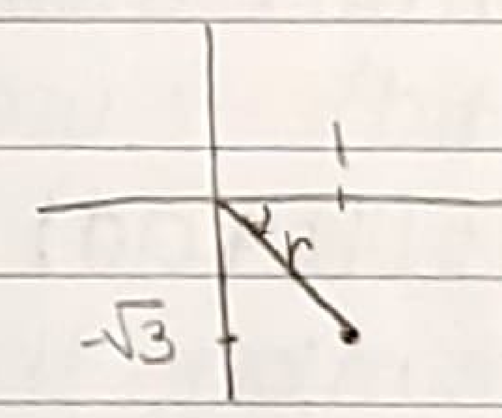
june

$z_1 = r_1 \operatorname{cis}(\theta_1)$
$z_2 = r_2 \operatorname{cis}(\theta_2)$ > mult. ⟶ $z_1 z_2 = r_1 r_2 \underbrace{\operatorname{cis}(\theta_1)\operatorname{cis}(\theta_2)}_{\text{expand}}$ ⟶

⟶ $\operatorname{cis}(\theta_1) \cdot \operatorname{cis}(\theta_2) =$
$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$
$= \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 + i(\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2)$
↳ because $(i\sin\theta)(i\sin\theta) = i^2 \sin \rightarrow i^2 = -1 \rightarrow -\sin\theta_1 \sin\theta_2$
↳ angle sum identities
$= \cos(\theta_1 + \theta_2) + i(\sin(\theta_1 + \theta_2))$
$= \operatorname{cis}\theta_1 \operatorname{cis}\theta_2 = \operatorname{cis}(\theta_1 + \theta_2)$ ⟶ $\underline{\operatorname{cis}\theta = e^{i\theta}}$ euler's formula

ⓔⓧ find polar coordinate for $z = 1 - i\sqrt{3}$ 02 June 2023

real part = 1 $x = 1$
imaginary part = $-\sqrt{3}$ $y = -\sqrt{3}$ ⟶



$|z|$ = modulus of $z = r$
↳ how far z is from the origin
$r = \sqrt{1^2 + (-\sqrt{3})^2} = 2$
$r = 2$ $\theta = \arg(z)$ ⟶ principal argument of z
↳ $\operatorname{Arg}(z) \in (-\pi, \pi)$

$\cos\theta = \frac{x}{r} = \frac{1}{2}$
$\cos = \frac{1}{2} = \cos\frac{\pi}{3}$ ⟶ $\theta = (-)\pi/3$ ⟶ $\operatorname{Arg}(z) = -\frac{\pi}{3}$

we know that $z = re^{i\theta} = r\operatorname{cis}(\theta)$
$z = 2e^{i(-\pi/3)}$ or $2e^{-i\pi/3}$