june

$z_1 = r_1 \operatorname{cis}(\theta_1)$

$z_2 = r_2 \operatorname{cis}(\theta_2)$ $\xrightarrow{\text{mult.}}$ $z_1 z_2 = r_1 r_2 \underbrace{\operatorname{cis}(\theta_1)\operatorname{cis}(\theta_2)}_{\text{expand}}$

$\longrightarrow \operatorname{cis}(\theta_1) \cdot \operatorname{cis}(\theta_2) =$

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$= \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 + i(\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2)$

↳ because $(i\sin\theta)(i\sin\theta) = i^2 \sin \rightarrow i^2 = -1 \rightarrow -\sin\theta_1 \sin\theta_2$

↳ angle sum identities

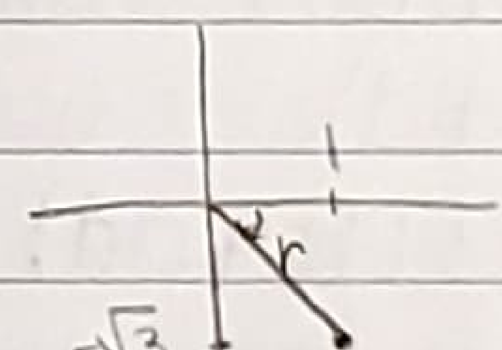
$= \cos(\theta_1 + \theta_2) + i(\sin(\theta_1 + \theta_2))$

$= \operatorname{cis}\theta_1 \operatorname{cis}\theta_2 = \operatorname{cis}(\theta_1 + \theta_2) \rightarrow \underline{\operatorname{cis}\theta = e^{i\theta}}$ euler's formula

ex find polar coordinate for $z = 1 - i\sqrt{3}$ 02 June 2023

real part = 1 $x = 1$

imaginary part = $-\sqrt{3}$ $y = -\sqrt{3}$ $\rightarrow$



$|z|$ = modulus of $z = r$

↳ how far z is from the origin

$r = \sqrt{1^2 + (-\sqrt{3})^2} = 2$

$r = 2$ $\theta = \arg(z) \rightarrow$ principal argument of z

↳ $\operatorname{Arg}(z) \in (-\pi, \pi)$

$\cos\theta = \frac{x}{r} = \frac{1}{2}$

$\cos = \frac{1}{2} = \cos\frac{\pi}{3} \longrightarrow \theta = (-)\pi/3 \rightarrow \operatorname{Arg}(z) = -\frac{\pi}{3}$

we know that $z = re^{i\theta} = r\operatorname{cis}(\theta)$

$z = 2e^{i(-\pi/3)}$ or $2e^{-i\pi/3}$

Practice with multiplication: and ÷

z_1, z_2 : complex numbers
In standard form: $z_1 = a+ib$
$z_2 = c+id$
In polar form: $z_1 = r_1 e^{i\theta_1}$
$z_2 = r_2 e^{i\theta_2}$
$e^{i\theta} = \text{cis}\,\theta = \cos\theta + \sin\theta$

if adding them:
- in standard → $(a+c) + i(b+d)$
- in polar form → not compatible

↳ same form if subtracting

- to multiply them in standard:

$z_1 z_2 = (a+ib)(c+id)$ → use foil method to distribute
$= ac - bd + i(bc+ad)$
↳ remember that the − comes from $i^2 = -1$

- to multiply them in polar form:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

- to divide using standard form:

$\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{\underbrace{(c+id)(c-id)}_{\text{conjugate of } z_2}}$

$z_1 = c+id$
$\bar{z} = c-id$

$= c^2 + (id)(-id) + (c)(-id) + (c)(id) = c^2 + d^2 = \frac{ac+bd+i(bc-ad)}{c^2+d^2}$

$-i^2 d^2$
↓
1

$= \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}$

▶ to divide using polar form:

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

(divide modulus, subtract argument)

ex) $z = 1 - i\sqrt{3}$, find z^5 and $1/z^2$

$r = 2 \quad \theta = -\pi/3 \rightarrow$ polar form: $z = 2e^{i(-\pi/3)}$

▷ so, multiply the modulus and add argument to find z^5

$= 2^5 e^{i5(-\pi/3)}$

$= 32 e^{-i\frac{5\pi}{3}} \longrightarrow \theta = -\frac{5\pi}{3} + 2\pi = \pi/3$

$\boxed{= 32e^{i\pi/3}}$

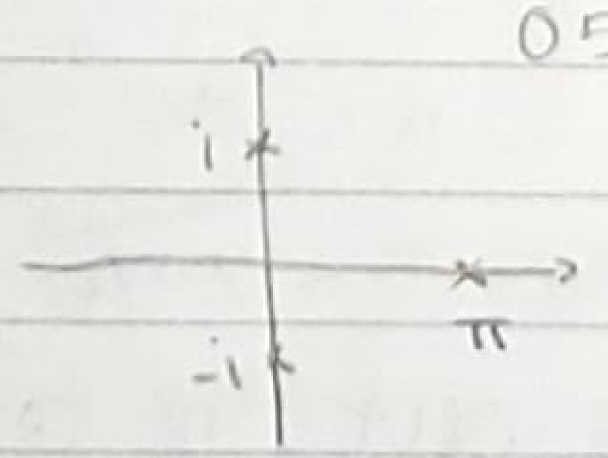
▷ $\frac{1}{z^2} = \frac{1 \cdot e^{i0}}{2^2 e^{i2(-\pi/3)}} = \frac{1}{2^2} e^{i(0 - 2(-\pi/3))} = \frac{1}{4} e^{i\frac{2\pi}{3}}$

For i

05 June 2023

$i = r\,\text{cis}(\theta) \rightarrow r = 1 \quad \theta = \pi/2 \qquad i = 1\,\text{cis}(\pi/2)$

$-i \rightarrow r = 1 \quad \theta = -\pi/2 \qquad -i = \text{cis}\, -\pi/2$



$-8i = 8\,\text{cis}(-\pi/2)$ remember r can't be negative

$\pi = \pi\,\text{cis}(0)$

Vectors

- similar to numbers (1, 2, 3, 4 ...)
  - vectors: group in pairs → $\underbrace{(1,2) \quad (5,-4) \quad (-1,\sqrt{2})}_{\text{2-component vectors}}$
- a vector represents a point on the plane