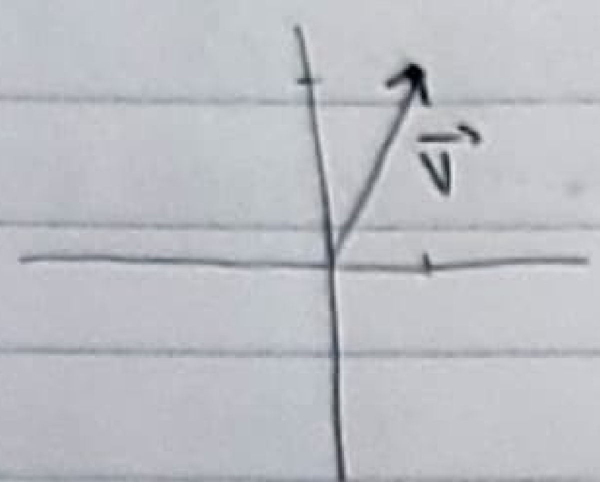
ex $\vec{v} = (1,2)$



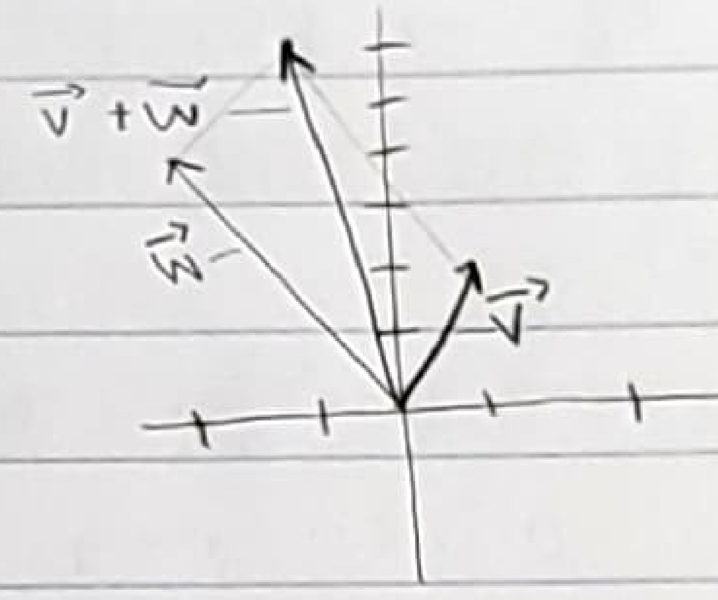
(1,2) is the coordinate point, to draw $\vec{v}$
draw → from origin to point

- vectors are used to describe <u>quantity</u> and <u>direction</u>

Operations of vectors

$\vec{v} = (1,2)$ $\vec{w} = (-2,4)$

x-comp, y-comp



makes a parallelogram

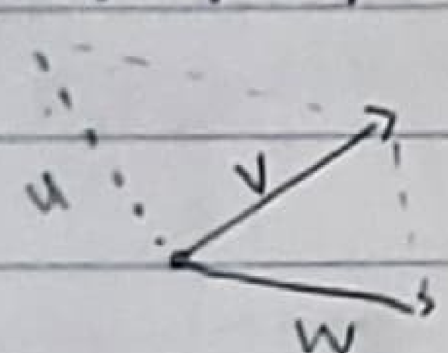
addition $\vec{v} + \vec{w} = (1+-2,\ 2+4) \rightarrow (-1,6)$

↳ just add the components

★ $\vec{v}+\vec{w}$ can be computed by adding components and replicated by the parallelogram rule ★

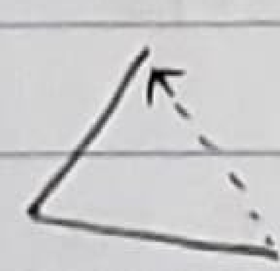
Subtraction $\vec{v} = (1,2)$ $\vec{w} = (2,3)$

$\underbrace{\vec{v} - \vec{w}}_{\vec{u}} = (-1,-1)$



parallelogram rule again since u + w = v

so connect tip to tip
then move to origin



06 June 2023

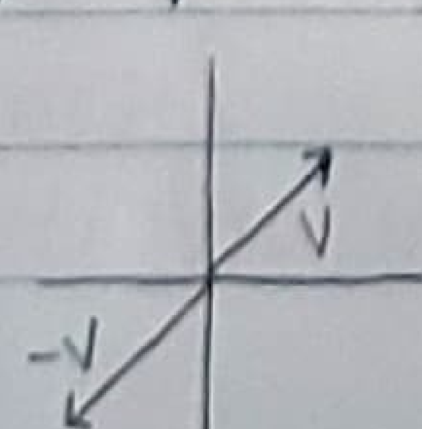
- $\vec{v}$ can be written as $\langle x, y \rangle$
- all vectors that have the same x, y coordinates are equivalent to each other

Vector zero

$\vec{0} = \langle 0,0 \rangle$

$\vec{v} + \vec{0} = \vec{v}$

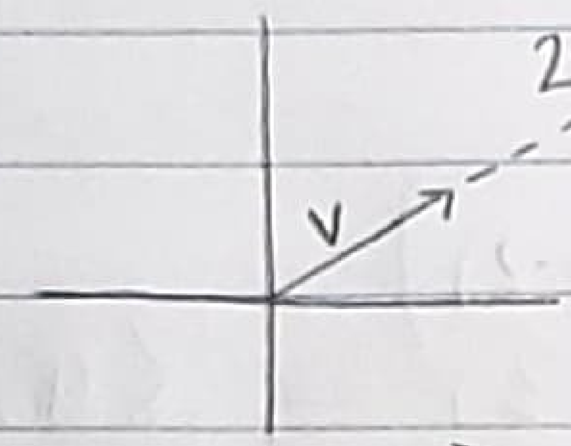
$\vec{0} - \vec{v} = -\vec{v}$ (reflected about the origin)



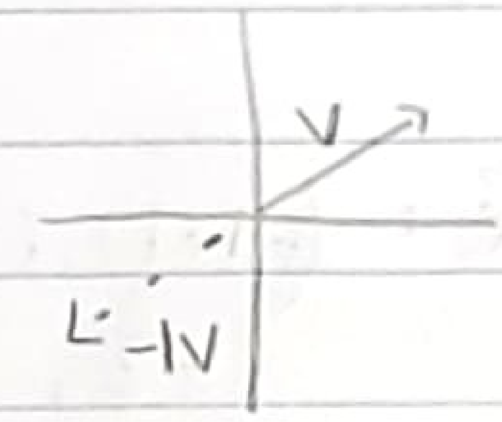
Operations of vectors continued

- Scaling
  - stretch the vector out by a said amount

(ex) scale $\vec{v}$ by 2



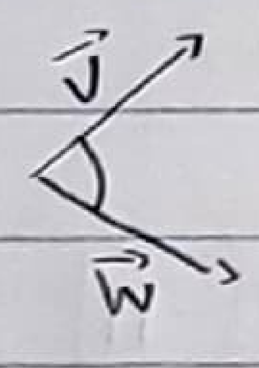
by -1



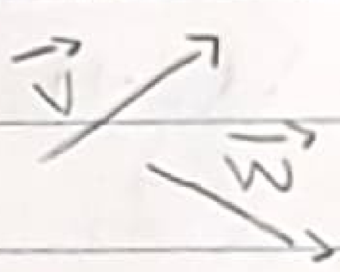
$\triangleright$ • $\vec{v} = \langle x, y \rangle \longrightarrow c\vec{v} = \langle cx, cy \rangle$ or $c\langle x, y \rangle$

(c: scale factor)

- Angles
  - remember that the angle is not sensative to position.



will have the same angle as



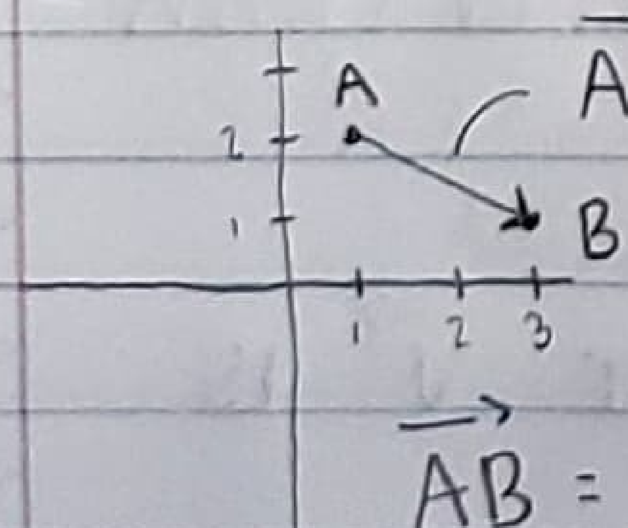
because you can move the vectors around so they touch

  - to find an angle between two vectors, we use the dot product

mult. $\triangleright$ $\longrightarrow$ $\left.\begin{array}{l}\vec{v} = \langle a, b \rangle \\ \vec{w} = \langle c, d \rangle\end{array}\right\}$ $\vec{v} \cdot \vec{w} = \underbrace{ac + bd}_{\text{results in a \# not a } \vec{v}}$

  - the dot product $\rightarrow$ angle
  - the cross product $\rightarrow$ area

(ex)

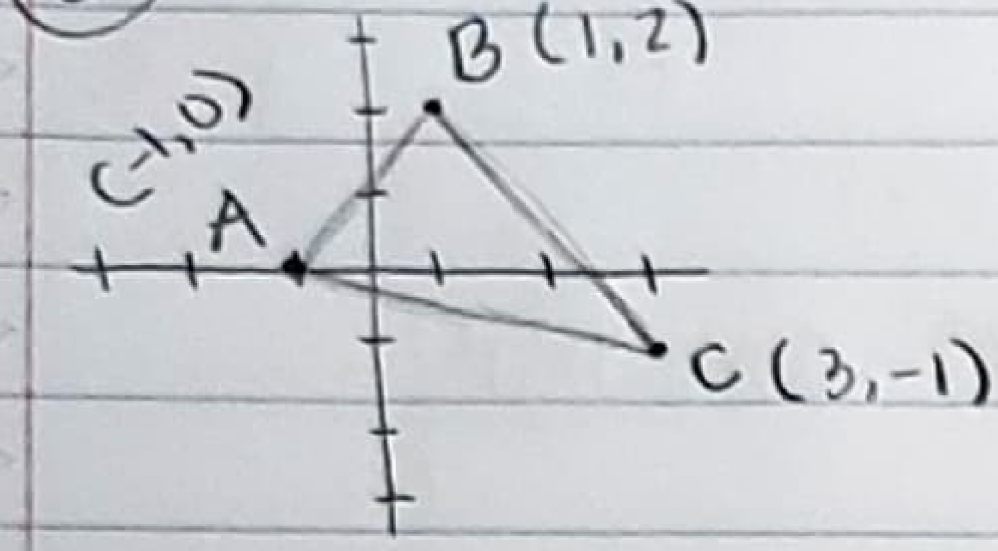


$\vec{AB}$ (vector that connects points A and B)

$\vec{AB} = \langle 2, -1 \rangle$

$\vec{AB} = \underbrace{\langle x_B - x_A, y_B - y_A \rangle}_{\text{similar to finding slope, just coordinate form}}$

(ex)



Find: $\overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AB} - 2\overrightarrow{AC}$

$\overrightarrow{BC} \cdot \overrightarrow{CA}$

$\overrightarrow{AB} + \overrightarrow{BC}$: $((1+1),(2-0)) + ((3-1),(-1-2))$

$(2, 2) + (2, -3) = \boxed{\langle 4, -1 \rangle}$

$\overrightarrow{AB} - 2\overrightarrow{AC}$: $(2,2) - 2\langle (3+1), (-1-0) \rangle$

$\langle 2,2 \rangle - 2\langle 4,-1 \rangle$

$\langle 2,2 \rangle - \langle 8,-2 \rangle = \boxed{\langle -6, 0 \rangle}$

$\overrightarrow{BC} \cdot \overrightarrow{CA}$: $\langle (3-1),(-1-2) \rangle \cdot \langle (-1-3),(0+1) \rangle$

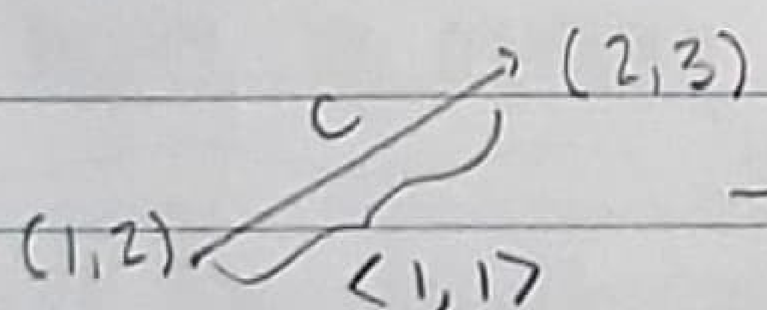
$\langle \underset{a}{2}, \underset{b}{-3} \rangle \cdot \langle \underset{c}{-4}, \underset{d}{1} \rangle = 2(-4) + (-3)1$

$-8 + -3 = \boxed{-11}$

Angle between two vectors

08 June 2023

- the length of $\vec{v}$ is denoted by $\|\vec{v}\|$
  - use pythagorean theorum



$\longrightarrow 1^2 + 1^2 = c^2 = 2 = c^2 \longrightarrow c = \sqrt{2}$

  - If $\vec{v} = \langle a,b \rangle$ then $\|\vec{v}\| = \sqrt{a^2+b^2}$
- the length is also called magnitude, modulus, or norm
- $\|\vec{v}\|$ can be written as $|\vec{v}|$
- Unit vector
  - the unit vector in direction of vector $\vec{v}$ is

$$e_{\vec{v}} = \frac{\vec{v}}{\|\vec{v}\|}$$

★ we want the length = 1 ★