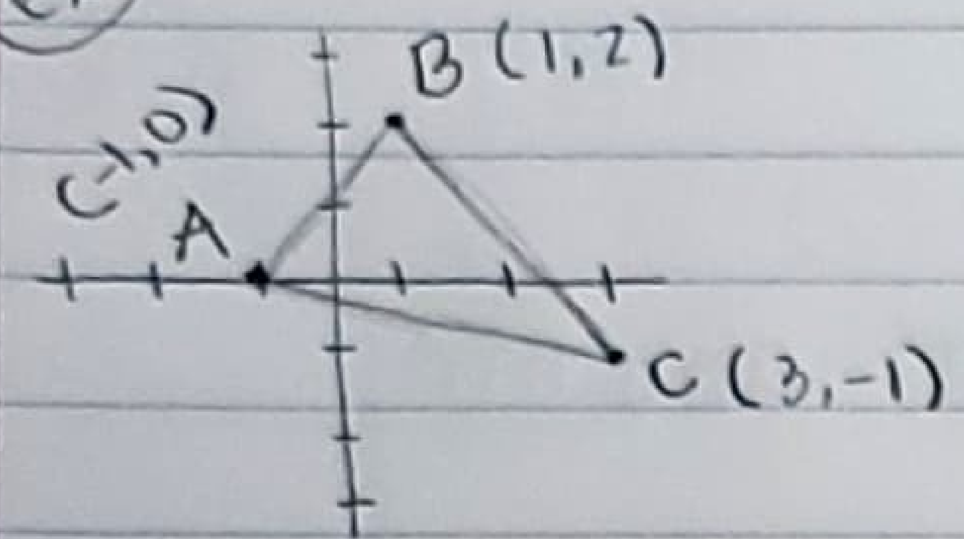
ex



Find: $\overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AB} - 2\overrightarrow{AC}$

$\overrightarrow{BC} \cdot \overrightarrow{CA}$

$\overrightarrow{AB} + \overrightarrow{BC}$: $((1+1),(2-0)) + ((3-1),(-1-2))$

$(2, 2) + (2, -3) = \boxed{\langle 4, -1 \rangle}$

$\overrightarrow{AB} - 2\overrightarrow{AC}$: $(2,2) - 2\langle (3+1), (-1-0) \rangle$

$\langle 2,2 \rangle - 2\langle 4, -1 \rangle$

$\langle 2,2 \rangle - \langle 8, -2 \rangle = \boxed{\langle -6, 0 \rangle}$

$\overrightarrow{BC} \cdot \overrightarrow{CA}$: $\langle (3-1), (-1-2) \rangle \cdot \langle (-1-3), (0+1) \rangle$

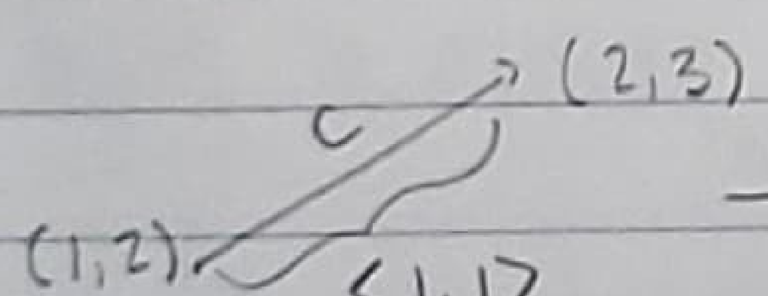
$\langle \underset{a}{2}, \underset{b}{-3} \rangle \cdot \langle \underset{c}{-4}, \underset{d}{1} \rangle = 2(-4) + (-3)1$

$-8 + -3 = \boxed{-11}$

Angle between two vectors

08 June 2023

- the length of $\vec{v}$ is denoted by $\|\vec{v}\|$
  - use pythagorean theorum



$\longrightarrow 1^2 + 1^2 = c^2 = 2 = c^2 \rightarrow c = \sqrt{2}$

  - If $\vec{v} = \langle a,b \rangle$ then $\|\vec{v}\| = \sqrt{a^2+b^2}$
- the length is also called magnitude, modulus, or norm
- $\|\vec{v}\|$ can be written as $|\vec{v}|$
- Unit vector
  - the unit vector in direction of vector $\vec{v}$ is

$$e_{\vec{v}} = \frac{\vec{v}}{\|\vec{v}\|}$$

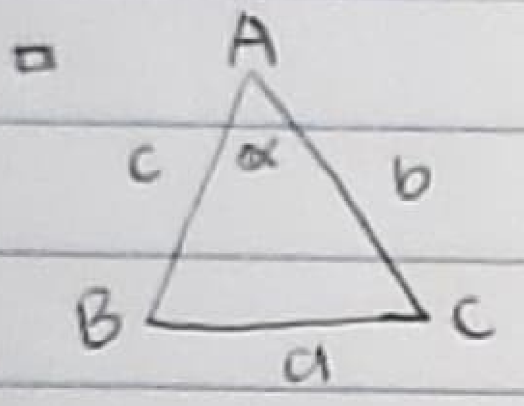
★ we want the length = 1 ★

- If the length is >1 or <1, we either scale the vector up or down to get the length equal to 1

- Angle formula:
  - the angle $\theta \in [0, \pi]$ between $\vec{u}$ and $\vec{v}$ is given by:

$$\cos\theta = e_{\vec{u}} \cdot e_{\vec{v}}$$

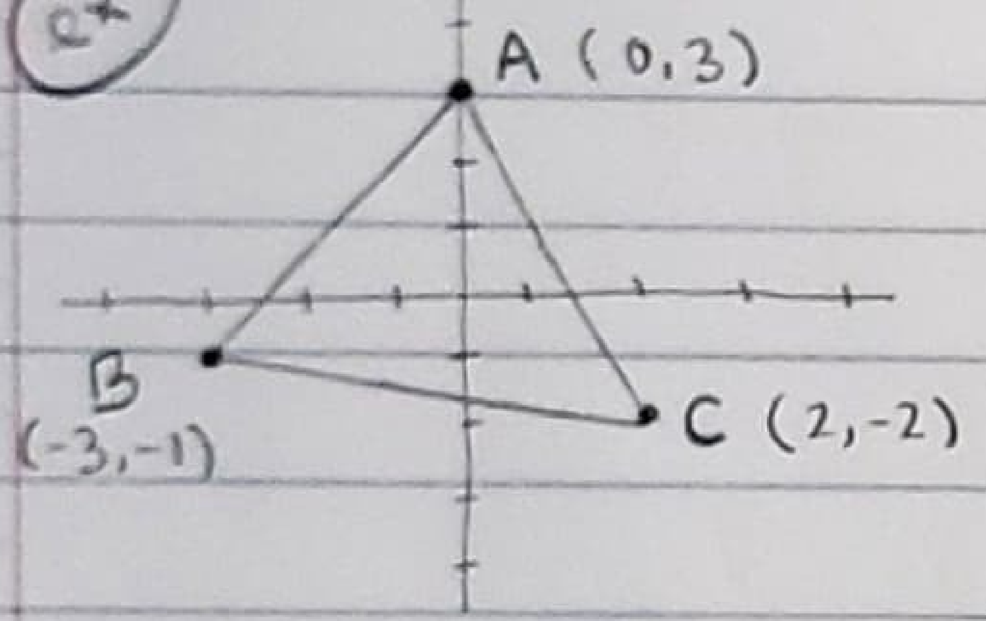
  - this formula comes from the Law of Cosines



$$\rightarrow \cos\alpha = \frac{c^2 + b^2 - a^c}{2bc}$$

  - relates the geometry with the algebra

ex



Find length of AB, BC, CA:

AB: $(-3,-1)$
$-(0,3)$
$\langle -3,-4\rangle \rightarrow -3^2 + 4^2 = 9 + 16 = 25$ $\quad \|\vec{AB}\| = \sqrt{25} = 5$

BC: $(2,-2)$
$-(-3,-1)$
$\langle 5,-1\rangle \rightarrow 5^2 + -1^2 = 25 + 1 = 26$ $\quad \|\vec{BC}\| = \sqrt{26}$

CA: $(0,3)$
$-(2,-2)$
$\langle -2,5\rangle \rightarrow -2^2 + 5^2 = 4 + 25 = 29$ $\quad \|\vec{CA}\| = \sqrt{29}$

$\llcorner$ AC: $\langle 2,-5\rangle$

Find the angle at A,B,C:

$\cos\theta = e_{\vec{u}} \cdot e_{\vec{v}}$ or $e_{\vec{v}} = \vec{v}/\|\vec{v}\|$

A: $e_{\vec{AB}}: \dfrac{\langle -3,-4\rangle}{5} = \langle -3/5, -4/5\rangle$

$e_{\vec{AC}} = \dfrac{\langle 2,-5\rangle}{\sqrt{29}} = \langle 2/\sqrt{29}, -5/\sqrt{29}\rangle$

$\langle -3/5, -4/5\rangle \cdot \langle 2/\sqrt{29}, -5/\sqrt{29}\rangle = (-3/5)(2/\sqrt{29}) + (-4/5)(-5/\sqrt{29})$

$= \left(-6/5\sqrt{29}\right) + \left(20/5\sqrt{29}\right) = 14/5\sqrt{29} = 0.52$

$\cos A = 0.52 \rightarrow \arccos(0.52) = \boxed{58.67°}$

B: $e_{\vec{BA}}: \dfrac{\langle 3,4\rangle}{5} = \langle 3/5, 4/5\rangle$

$e_{\vec{BC}}: \dfrac{\langle 5,-1\rangle}{\sqrt{26}} = \langle 5/\sqrt{26}, -1/\sqrt{26}\rangle$

$\langle 3/5, 4/5\rangle \cdot \langle 5/\sqrt{26}, -1/\sqrt{26}\rangle = (15/5\sqrt{26}) + (-4/5\sqrt{26})$

$11/5\sqrt{26} = 0.43 \rightarrow \cos B = 0.43$

$\arccos(0.43) = \boxed{64.5°}$