Lecture 9

Monday, April 17, 2023 8:32 AM

* Questions

* Solving equations involving the logarithm

- $\log_a x = b \iff x = a^b$
- $\log_a x = \log_a y \iff x = y$

Ex

1) $\log(3x-1) = \log(4-x)$

2) $\ln(8-x^2) = \ln(2-x)$

3) $\log_{\frac{1}{2}}(2x-1) = -3$

4) $\ln(x^2) = (\ln x)^2$

* Solving inequalities involving logarithm:

- If $a>1$.

$$\log_a x \begin{matrix} \geqslant \\ > \\ < \\ \leqslant \end{matrix} \log_a y \iff x \begin{matrix} \geqslant \\ > \\ < \\ \leqslant \end{matrix} y$$

- If $0<a<1$.

$$\log_a x \begin{matrix} \geqslant \\ > \\ < \\ \leqslant \end{matrix} \log_a y \iff x \begin{matrix} \leqslant \\ < \\ > \\ \geqslant \end{matrix} y$$

Ex

1) $\log_{\frac{1}{2}}(x^2+1) \leq -1$

2) $x(\ln x - 1) > 0$

3) $\log_{\frac{1}{2}}(x^2-5x+6) > -1 = \log_{\frac{1}{2}}(\frac{1}{2})^{-1} = \log_{\frac{1}{2}}(2)$

We need $x^2-5x+6>0$ and $x^2-5x+6<2$.

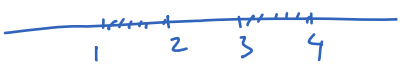
That is $\begin{cases} (x-2)(x-3)>0 \\ (x-1)(x-4)<0 \end{cases}$

x		2		3	
$x-2$	$-$	0	$+$	$\|$	$+$
$x-3$	$-$	$\|$	$-$	0	$+$
$(x-2)(x-3)$	$+$	0	$-$	0	$+$

We need $x \in (-\infty, 2) \cup (3, \infty)$

x		1		4	
$x-1$	$-$	0	$+$	$\|$	$+$
$x-4$	$-$	$\|$	$-$	0	$+$
$(x-1)(x-4)$	$+$	0	$-$	0	$+$

We need $x \in (1, 4)$



Therefore, $x \in (1,2) \cup (3,4)$.