Final exam: Some problems for review

The exam will be held at the regular classroom (Badgley Hall 146) from 8 AM to 10 AM on Monday, June 12, 2023. The material covered is Section 8.7 - 9.5. It is a closed book exam. A scientific calculator is allowed. Graphing/ programmable/ transmittable calculators are not allowed. You can bring the conic section formula sheet (free of any handwriting). A 4” x 6” handwritten single-sided note card is also allowed. The following formulas will be provided on the exam.

$$\begin{aligned} e^x &= \sum_{k=0}^{\infty} \frac{x^k}{k!} \\ \sin x &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} \\ \cos x &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} \\ \ln(x+1) &= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k \end{aligned}$$

You should review the homework problems, the examples given in the textbook and in the lectures. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Find the Taylor series of some simple functions.
- Determine the n’th Taylor polynomial of a function.
- Sketch a curve by using the parametric equations to plot points.
- Find a tangent line and the length of a parametric curve.
- Find the area bounded by a parametric curve.
- Sketch a curve given by a polar equation.
- Find the tangent line and length of a polar curve.
- Find the area bounded by a curve and two rays.
- Find the equation of a conic section in Cartesian coordinates and polar coordinates.

Additional problems to practice:

1) Find the Maclaurin series (i.e. the Taylor series centered at 0) of the functions

   (a) $\sin(\pi x)$

   (b) $e^x + e^{2x}$

   (c) $e^x + 2e^{-x}$

2) When x is close to 0, approximate the function $f(x) = \sin(e^x - 1)$ by a polynomial of degree 2. Then use this approximation to approximate the value of $f(0.2)$.

3) Sketch the curve

$$x = 1 - t, \quad y = 1 - t^2, \quad t \in [-2, 2]$$

by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

4) Find an equation of the tangent to the curve

$$x = t - \frac{1}{t}, \quad y = 1 + t^2$$

at the point $(0, 2)$.

5) Find the length of the curve

$$x = e^t + e^{-t}, \quad y = 5 - 2t, \quad t \in [0, 3]$$

6) Consider a point on the plane with the Cartesian coordinates $(-2, 2\sqrt{3})$. Find two other pairs of polar coordinates of this point, one with $r > 0$, $0 < \theta < 2\pi$ and one with $r < 0$, $-4\pi < \theta < -2\pi$.

7) Sketch the polar curve $r = -2\sin\theta$.

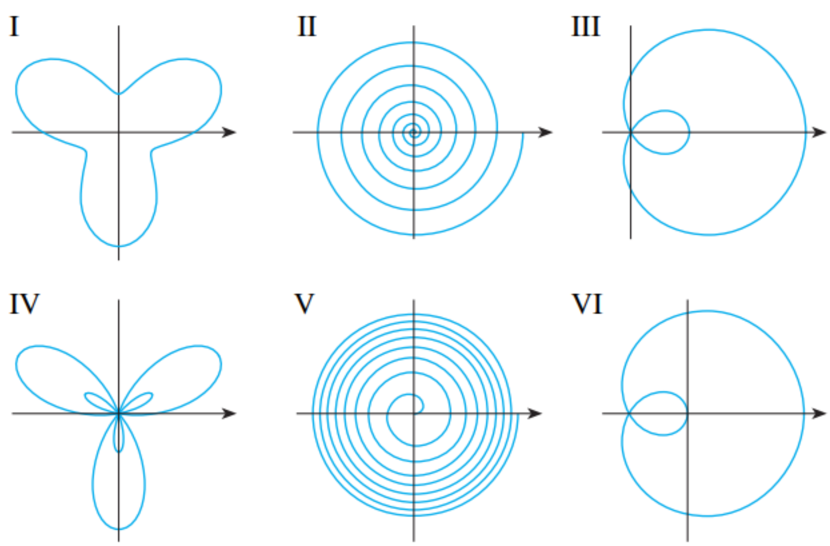
8) Do Problem 46 on page 529:

46. Match the polar equations with the graphs labeled I–VI. Give reasons for your choices. (Don't use a graphing device.)

(a) $r = \sqrt{\theta}$, $0 \leqslant \theta \leqslant 16\pi$ (b) $r = \theta^2$, $0 \leqslant \theta \leqslant 16\pi$

(c) $r = \cos(\theta/3)$ (d) $r = 1 + 2\cos\theta$

(e) $r = 2 + \sin 3\theta$ (f) $r = 1 + 2\sin 3\theta$

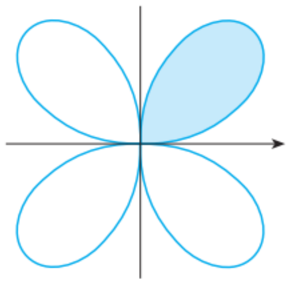


9) Find the area of the shaded region in the flower figure below.

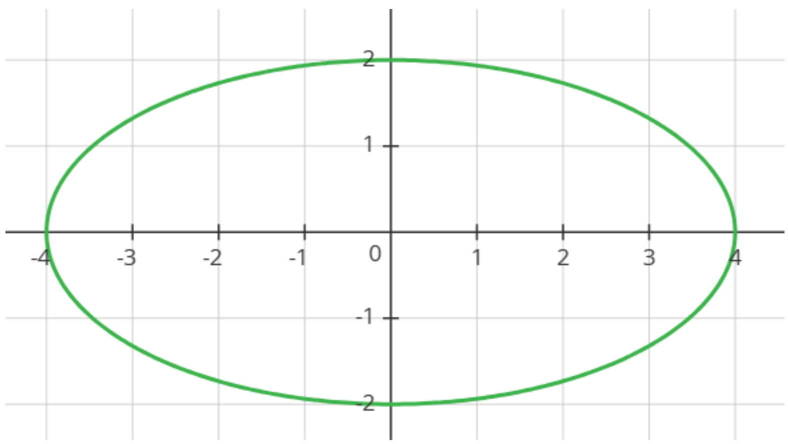
10) Consider an ellipse given in the picture.

(a) Find the coordinates of the foci, the equation of the directrices, and the eccentricity.

(b) Choose the pole and the polar axis of a polar coordinate system. Then write an equation of the ellipse in polar coordinates.



$r = \sin 2\theta$



11) Consider a conic curve given by the polar equation

$$r = \frac{12}{3 - 9\cos\theta}$$

(a) Find the eccentricity.

(b) Identify the conic.

(c) Give an equation of the directrix.

(d) Sketch the conic and specify from the picture where the pole and polar axis of the polar coordinate system are.