Lecture 1

Thursday, March 30, 2023 4:16 PM

* Introduction : instructor and class members

* Syllabus : grading components, software, attendance, late policy.

Calc I, II : learned about functions (limit, derivative, integral).

What can we do with those tools ?

- Compute rate of change, speed
- Check functions' properties such as : increasing / decreasing, (monotonicity)

asymptotic behavior (limit), critical points, min/max,

area of a region, volume of a solid of revolution.

Calc III is built on Calc I, II. It has two parts :

- Series (Chapter 8) : infinite sum
- Curves (Chapter 9) : parametrization, length, enclosed area, tangent

We'll also cover some basics about vectors (a little bit of Chapter 10).

* Series : a series is an infinite sum.

Ex : $1+2+3+4+5+\cdots = \infty$

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots = 1$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = \infty$$

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$: Taylor expansion of the function $\frac{1}{1-x}$.

$$\underbrace{1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = e^x}$$: Taylor expansion of e^x.

decomposition of e^x into normal modes

Analogy: optical dispersion

white light is "decomposed" into different colors, each corresponding to a range of wave length.

A series can be seen as the limit of a sequence:

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} \right)$$

* Sequence $a_1, a_2, a_3, \ldots$

$b_5, b_6, b_7, \ldots$

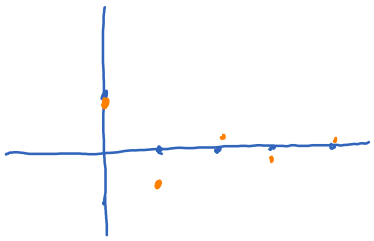
Ex $1, 2, 3, 4, \ldots$ $a_n = n$, $n \geq 1$

$2, 4, 6, 8, \ldots$ $a_n = 2n$, $n \geq 1$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ $a_n = \frac{1}{2^n}$, $n \geq 0$

$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots$ $a_n = \frac{1}{2^n}(-1)^n$, $n \geq 0$

Visualize



On Mathematica, use the command ListPlot.

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a = Table [ (-1)^n/2^n, {n, 0, 10}]
ListPlot [a].
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