Lecture 10

Thursday, April 20, 2023 1:47 AM

* Questions...

Integral Test:

If $a_n = f(n)$ and f is a nonnegative function and increasing on some interval $[M, \infty)$ then

$$\sum a_n < \infty \quad \text{iff} \quad \int_M^\infty f(x)\,dx < \infty$$

This convergence test works very nicely on the kind of series called the

p-series: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$

$p=1$: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ (harmonic series)

$p=2$: $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

$p=\frac{1}{2}$ $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$

The series $\sum \frac{1}{n^p}$ converges iff $p > 1$. If $p \le 1$, it diverges.

Ex Test the convergence / divergence of

(a) $\displaystyle\sum \frac{1}{n^2+1}$

(b) $\displaystyle\sum \frac{1}{n^3-n}$

(c) $\displaystyle\frac{1}{\ln n}$

(d) $\displaystyle\frac{1}{n \ln n}$

Alternating series test

An alternating series is a series of the form

$$b_1 - b_2 + b_3 - b_4 + b_5 - \cdots$$

or $$-b_1 + b_2 - b_3 + b_4 - b_5 + \cdots$$

These series converge if $\{b_n\}$ is a decreasing sequence and $\lim b_n = 0$.

Analogy of the zipper.