5/1/23 Representing a Function as a Power Series

$\sum a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

A power series define a function on the interval of convergence.

ex. $\sum_{n=1}^{\infty} \frac{x^n}{n}$ is a function defined on $[-1,1)$

$\ln(x+1)$ vs $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$

these are the same thing

Advantages of Power Series Representation:

(1) Aproximation of the function a computer/ calculator

(2) Differentiate or integrate easily

(3) Simplify the function: $\ln(1+x) \approx x - \frac{x^2}{2}$ (return to this power series)

ex. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$ when $x \in (-1,1)$

*memorize this

plugged the $(-x)$ in

ex. $\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + x^4 - \dots = \sum_{n=0}^{\infty} (-x)^n = \boxed{\sum (-1)^n x^n}$

ex. $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + (-x^2)^4 + \dots$

$\sum_{n=0}^{\infty} (-x^2)^n = \sum (-1)^n (x^2)^n = \boxed{\sum (-1)^n x^{2n}}$

power series

ex. $\frac{2}{3+5x} = 2 \frac{1}{3+5x} = \frac{2}{3} \frac{1}{1+\frac{5}{3}x} = \frac{2}{3} \cdot \frac{1}{1-(-\frac{5}{3}x)} =$

$\frac{2}{3}\left(1 + (-\frac{5}{3}x) + (-\frac{5}{3}x)^2 + (-\frac{5}{3}x)^3 + \dots\right) = \sum_{n=0}^{\infty} \frac{2}{3}(-\frac{5}{3}x)^n =$

$\frac{2}{3} \sum_{n=0}^{\infty} (-\frac{5}{3})^n x^n = \boxed{\sum \frac{2}{3}(-\frac{5}{3})^n x^n}$