* Quiz on Thursday, ratio test, root test, radius and interval of convergence

Continue... 5/2/23

ex. $\dfrac{x+1}{x^2-3x+2} = \dfrac{x+1}{(x-2)(x-1)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x-1)} = \dfrac{A(x-1)+B(x+2)}{(x-2)(x-1)}$

$x+1 = A(x-1) + B(x-2)$

plug $x=1$: $2 = B(-1)$ $\quad B=-2$ $\qquad \dfrac{3}{(x-2)} + \dfrac{-2}{(x-1)}$

plug $x=2$: $3 = A$

Write power series for $\frac{-2}{x-1}$

$-1\left(\dfrac{-2}{x-1}\right) = 2\dfrac{1}{1-x} = 2(1+x+x^2+x^3+\dots) = 2\sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} 2x^n$

Write power series for $\frac{3}{x-2}$

$-1\left(\dfrac{3}{x-2}\right) = -3\dfrac{1}{2-x} = -\dfrac{3}{2}\dfrac{1}{1-\frac{1}{2}x} = \sum_{n=0}^{\infty}\left(-\dfrac{3}{2}\right)\left(\dfrac{1}{2}x^n\right) = \sum_{n=0}^{\infty}\left(-\dfrac{3}{2}\right)\dfrac{x^n}{2^n}$

Combine:

$\dfrac{x+1}{x^2-3x+2} = \sum_{n=0}^{\infty} 2x^n + \sum_{n=0}^{\infty}\left(-\dfrac{3}{2}\right)\dfrac{1}{2^n}x^n = \sum_{n=0}^{\infty}\left(2x^n + \left(-\dfrac{3}{2}\right)\dfrac{1}{2^n}x^n\right)$

* Side not: (adding power series)

$\sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty}(a_n+b_n)$ $\qquad$ * If index is matching (like $n=0$)

$a_0 + a_1 + a_2 + a_3 + \dots$

$+\ b_0 + b_1 + b_2 + b_4 + \dots$

$(a_0+b_0) + (a_1+b_1) + (a_2+b_2) + (a_3+b_3) + \dots$

$\sum_{n=0}^{\infty} x^n \overbrace{\left(2 + \left(-\dfrac{3}{2}\right)\dfrac{1}{2^n}\right)}^{a_n}$ *answer

ex. $\dfrac{2x-1}{x^2-x-2} = \dfrac{2x-1}{(x-2)(x+1)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)} = A(x+1) + B(x-2)$

$2x-1 = A(x+1) + B(x-2)$

plug $x=-1$: $-2-1 = B(-3)$ $\quad B=1$ $\qquad$ plug $x=2$: $3 = A(3)$

$-3 = B(-3)$ $\qquad A=1$

$$\frac{1}{x-2}+\frac{1}{x+1}$$

$$\frac{1}{x-2}=-\frac{1}{2-x}=-\frac{1}{2}\,\frac{1}{1-\frac{x}{2}}=\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)\frac{x^n}{2^n}$$

$$\frac{1}{x+1}=\frac{1}{1-(-x)}=\sum_{n=0}^{\infty}(-x)^n=\sum_{n=0}^{\infty}(-1)^n x^n$$

$$\frac{2x+1}{x^2-x-2}=\sum_{n=0}^{\infty}\left(\left(-\frac{1}{2}\right)\frac{1}{2^n}x^2+(-1)^n x^n\right)$$

$$=\sum_{n=0}^{\infty}\left(\left(-\frac{1}{2}\right)\frac{1}{2^n}+(-1)^n\right)x^n$$

Theorem:

radius of convergence

On the interval $(x_0-R,\ x_0+R)$, the power series $\sum a_n(x-x_0)^n$ has derivative $\sum a_n n(x-x_0)^{n-1}$ and antiderivative $\sum \frac{a_n}{n+1}(x-x_0)^{n+1}+C$.

ex. $\int_0^1 \frac{1}{x^3+8}\,dx$

$r=2$

$$\frac{1}{x^3+8}=\frac{1}{8}\,\frac{1}{1+\frac{x^3}{8}}=\frac{1}{8}\sum\left(-\frac{x^3}{8}\right)^n=\sum_{n=0}^{\infty}\frac{1}{8}\left(-\frac{1}{8}\right)^n x^{3n}$$

$$\int_0^1 \dots dx=\sum\frac{1}{8}\left(-\frac{1}{8}\right)^n x^{3n}=\sum_{n=0}^{\infty}\frac{1}{8}\left(-\frac{1}{8}\right)^n\frac{1}{3n+1}$$