Lecture 2

Tuesday, April 4, 2023 8:46 AM

* Questions ...

A series is an infinite sum. The terms of this sum form a sequence.

A sequence is an infinite indexed list $a_1, a_2, a_3, a_4, \dots$

The index doesn't have to start at 1. It can start at any integer, for example:

$$a_0, a_1, a_2, a_3, \dots$$

$$a_7, a_8, a_9, \dots$$

$$a_{-3}, a_{-2}, a_{-1}, a_0, a_1, \dots$$

One can view a sequence as a function $a : \mathbb{N} \to \mathbb{R}$

$$a(n) \equiv a_n \quad : \text{simplified notation}$$

* Notation:

$$\{a_n\} \qquad (a_n) \qquad (a_n : n \geq 1)$$

$$\{a_n\}_{n=1}^{\infty} \qquad (a_n)_{n=1}^{\infty} \qquad (a_n : n = 1, 2, 3, \dots)$$

$$\{a_n\}_{n \geq 1} \qquad (a_n)_{n \geq 1}$$

Ways to represent a sequence:

① By enumeration:

$1, 2, 3, 4, 5, \ldots$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$2, 4, 6, 8, 10, \ldots$

One needs to make sure that the pattern is clear so that the reader can fill in more terms themselves.

$1, 3, 6, 12, 16, \ldots$: not a good way to represent a sequence.

② By generic formula:

$$\left.\begin{array}{l} a_n = \frac{1}{n} \\ \\ a_n = 2^n \\ \\ \ldots \end{array}\right\} \text{given } n, \ a_n \text{ can be computed directly from a formula.}$$

③ By recursion formula:

$$\begin{cases} a_1 = 1, \ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$$

This sequence is called the Fibonacci sequence.

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

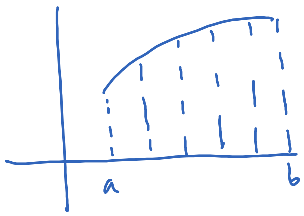
$a_6 = a_5 + a_4 = 5 + 3 = 8$

$a_7 = a_6 + a_5 = 8 + 5 = 13$

Sequence is usually used to approximate a quantity. For example,

$\pi \approx 3.14, 3.141, 3.1415, 3.14159, \ldots$

Another example is the Riemann sum:



$$R_n = \sum_{i=1}^{n} f(x_i) \frac{b-a}{n}$$

is an approximation of the true area under the curve.

For $\{a_n\}$ to be an approximation of a quantity L, we need

$$\lim_{n\to\infty} a_n = L$$

This means:

[For each $\varepsilon > 0$ (allowable error), there is an index N such that $|a_n - L| < \varepsilon$ for all $n > N$.]

Ex

- The sequence $1, -1, 1, -1, 1, -1, 1, -1, \ldots$ doesn't have a limit.

  In other words, the sequence diverges.

- The sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$ has a limit:

  $\lim_{n\to\infty} \frac{1}{n} = 0$. This sequence converges.