

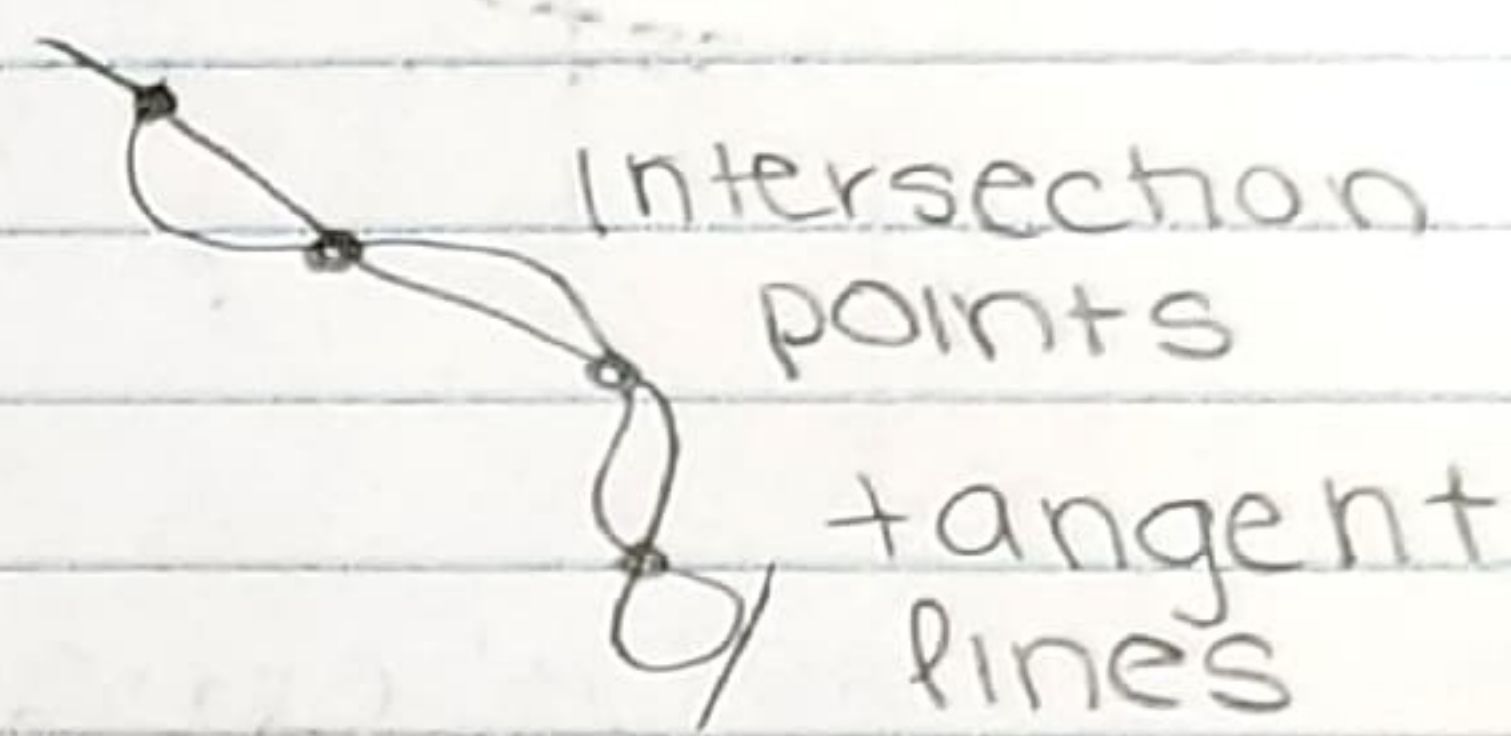
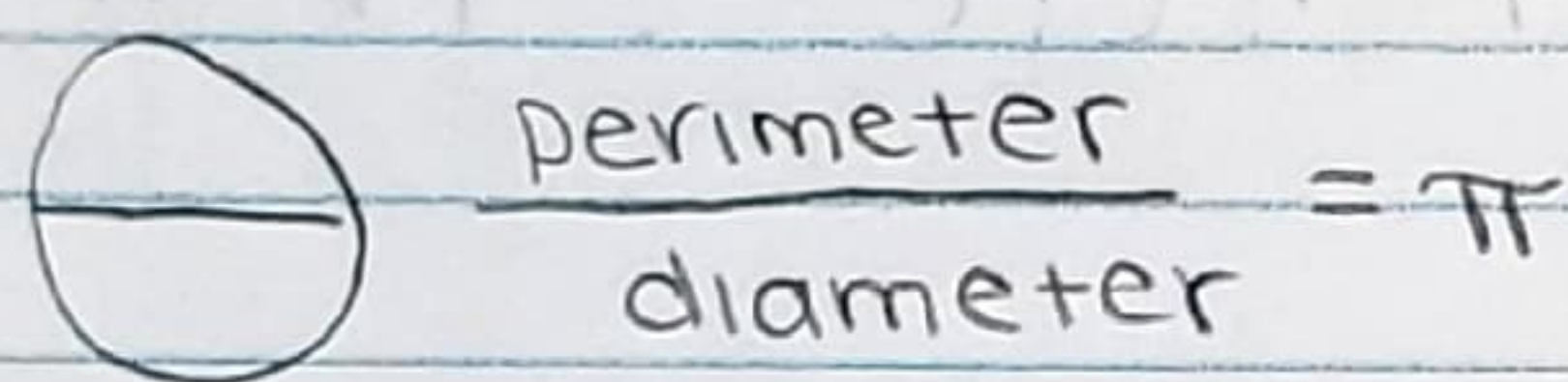
Curve & calculus on curves

5/12/23

- sine curve
- ellipse
- hyperbola
- circle
- parabola

The graph of a function is a curve.

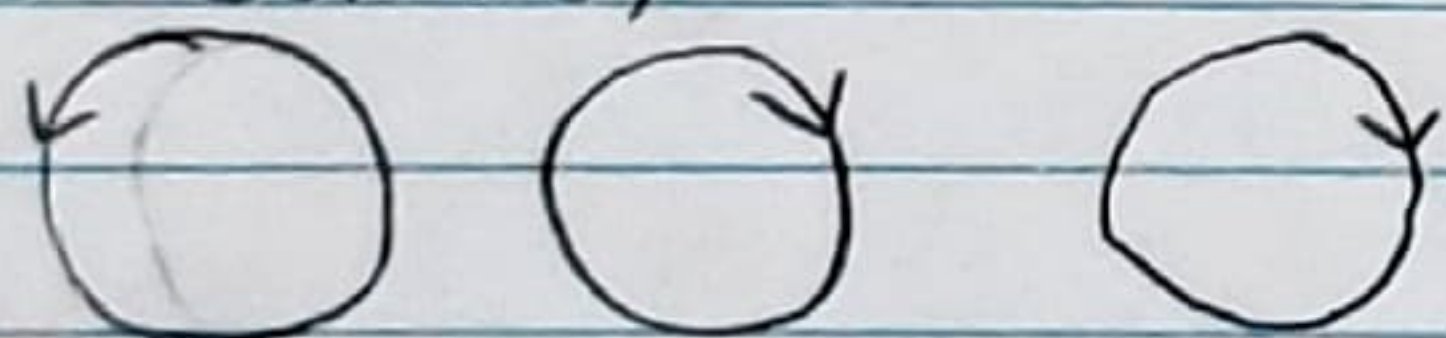
Purpose: use calculus to study curves



ex: compute length of a curve

Not all curves are graphs of functions.

*Parametrize a curve (a way to navigate a curve)



different parametrizations

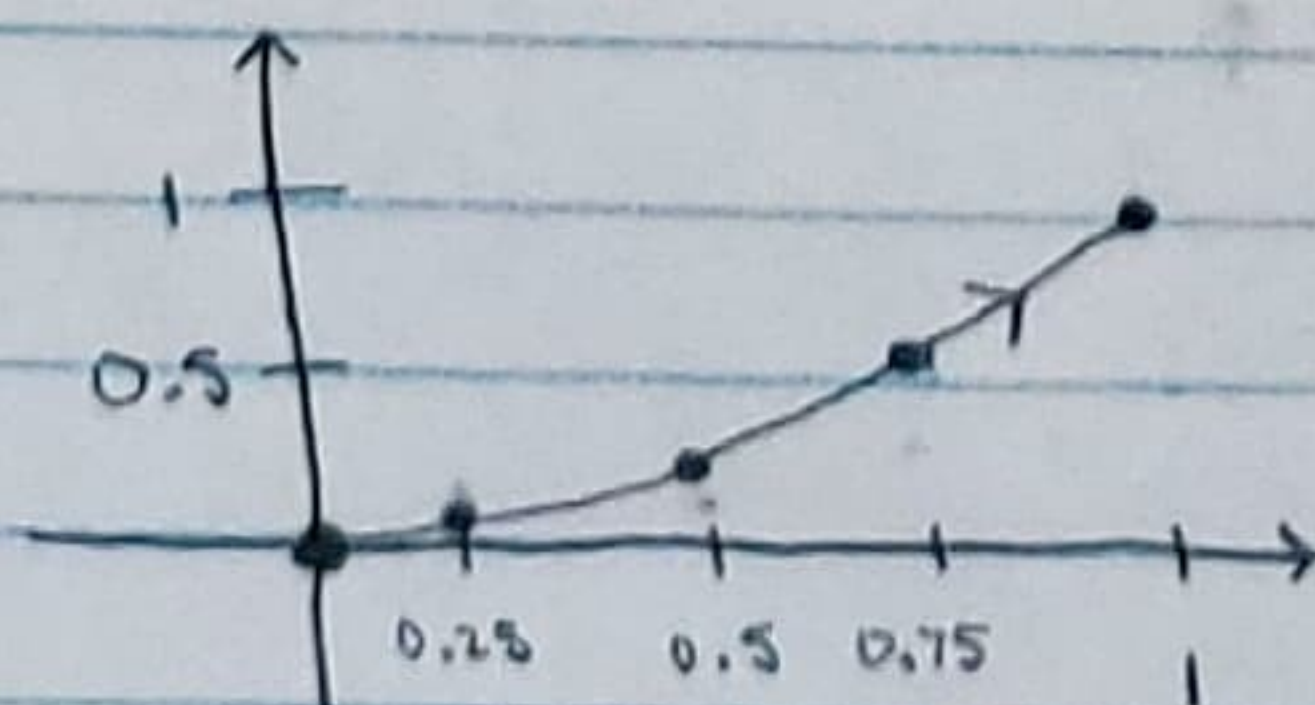
A parametrization of a curve is given in the form $\begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad a < t < b$

\leftarrow optional

ex. $\begin{cases} x = t & 0 \leq t \leq 1 \\ y = t^2 \end{cases}$

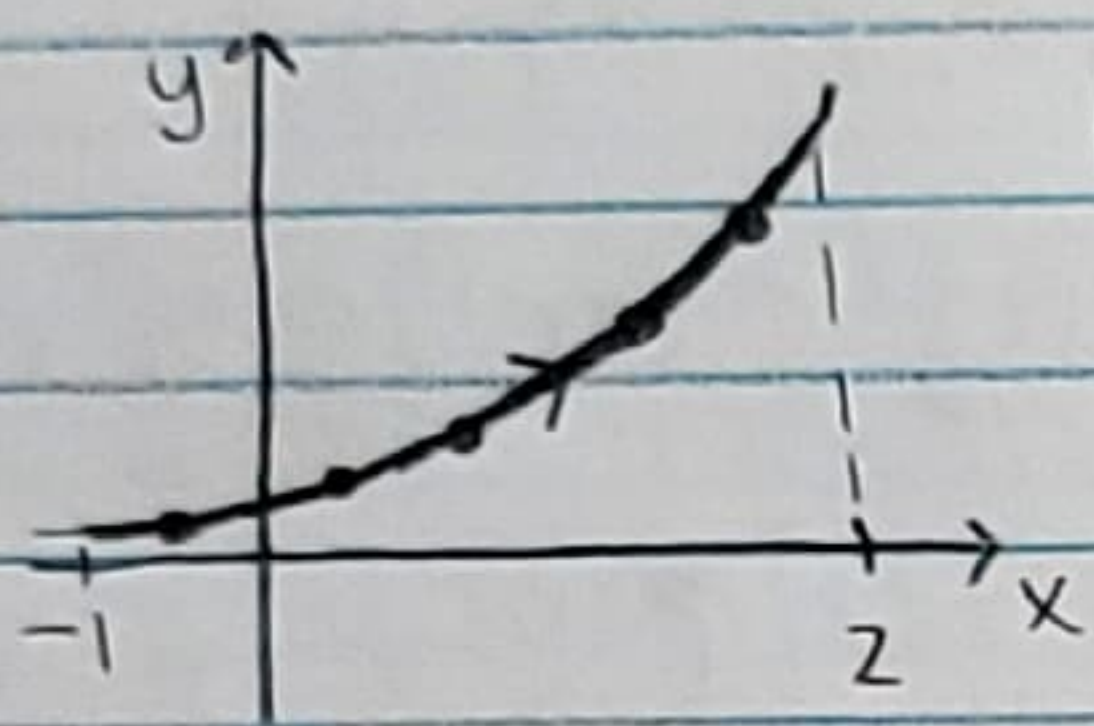
A value of t determines a pair (x, y)

t	x	y
0	0	0
0.25	0.25	0.0625
0.5	0.5	0.25
0.75	0.75	0.5625
1	1	1



$$t \rightarrow (x(t), y(t))$$

ex. Parametrize the graph of function
 $f(x) = e^x, x \in [-1, 2]$



① $\begin{cases} x = t & t \in [-1, 2] \\ y = e^t \end{cases}$ * Start slow, end fast

② $\begin{cases} x = \ln(t) & t \in [e^{-1}, e^2] \\ y = t \end{cases}$ * Start fast, end slow

*Mathematica

$x \quad y \quad \text{range}$
ParametricPlot[{{t, t^2}, {t, 0, 1}}